TRANSFORMATION OF THE WALSH HADAMARD PILOT SYMBOL TO AN EFFICIENT OPTIMAL PILOT SYMBOL

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Abstract: -

The OFDM is a modulation method vulnerable to non-linearities of source, channel or amplifier due to its high peak average ratio (PAPR). The distortion is exacerbated by the increase in the average OFDM signal strength, as more signals are impacted by non-linearity. In this article, we present Hadamard coded modulation (HCM), which employs the fast Walsh-Hadamard transform (FWHT) to modulate information in wireless optical systems as an alternative to OFDM. Due to its small PAPR this technique is shown to be better for high-medium optical power scenarios and can be used in two scenarios instead of OFDM: 1) in optical systems requiring higher optical power such as visible light communications, and 2) in wireless optical systems with lower mean power (BER) uncomplicated by average power. The power effectiveness of HCM can be increased without sacrificing information by eliminating some of the DC bias of the signal. This reduces the amplitude of the transmission signal and reduces the non-linearity of the signals. HCM can be connected to resist interlocking symbolic (ISI) effects via evenly spreading the interference to all symbols on dispersive channels. In section 1 we discuss about introduction, Section 2 tells about related work we build our proposed system using this related work. In section 3 we explain about proposed system, Section 4 tells about results what we got and final section 5 conclusion about our work.

1. INTRODUCTION

Wideband transmission with high spectral efficiency and peak mobility is necessary for next generation mobile radio communications. Walsh Hadamard Transform (WHT) is a precoding technique with having less complexity compared to the other existing power reduction techniques and also it can reduce PAPR considerably and results in no distortion. Interleaving can be applied on WHT to make the resulting signals resistant against ISI effects in frequency selective by uniformly distributing the interference over all symbols. The MIMO OFDM with interleaver using WHT is one of the best techniques to achieve this goal. The data are transmitted and received in MIMO OFDM system with many channel impairments like Multipath fading, AWGN, ISI, ICI, and Inter Antenna Interference (IAI).

WHT is a non-sinusoidal, orthogonal linear transform and can be implemented by a butterfly structure as in FFT. This means that applying WHT does not require the extensive increase of system complexity. WHT decomposes a signal into set of basic functions and achieve high spectral efficiency. These functions are Walsh functions, which are square waves with values of +1 or -1. The proposed Hadamard transform scheme may reduce the occurrence of the high peaks compared to the original OFDM system.

In OFDM system the total channel bandwidth is subdivided into number of sub streams, and all the sub channels constrain small bandwidth which are parallel and transmitted over the narrow sub channel. Now the symbol duration increases and ISI is caused by multipath fading. The sub channel orthogonality loss is due to channel variation with longer symbol duration. In such a case ICI may occur. As the transmission power improves, it leads to a larger effective capacity, wherein the energy consumption of the system also rises; therefore the larger power input results in the decline of energy efficiency.

The downside in the existing system is that the multichannel joint optimization problem in conventional MIMO-OFDM communication systems is converted into a multi target single-channel optimization problem by grouping all sub channels. Improving energy efficiency with a quality of the signal constraint remains a major drawback in MIMO-OFDM mobile multimedia system communication. There has been few analysis works related to the problem of optimizing energy efficiency under the various quality of the signal constraints in MIMO-OFDM mobile MIMOOFDM communication systems. The multichannel optimization problem is simplified using sub channels in various groups, which therefore forms a multi-target one channel optimization problem.

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The WHT is used in a number of applications, such as image processing, speech processing, filtering, and power spectrum analysis. Like the FFT, the WHT has a fast version, here introduced is a Fast Walsh-Hadamard transform (FWHT) as an alternative technique to FFT. The proposed method detects the transmitted data and deals with interferences, AWGN, channel fading and reduces complexity. The MIMO-OFDM system is an attractive method for high data rate wireless applications. OFDM systems provide long symbol duration by means of combining multiple low data-rate sub carriers. When the cyclic prefix is added, the ISI is completely removed. In MIMO OFDM, as the number of antennas increased the complexity is high and the bit error rate is more. When BER is focused, it affects the PAPR. The interleaver is introduced to manage the PAPR and the pilot insertion reduces the BER.

Hadamard matrices and the Fast Walsh Hadamard transform (FWHT) are the best tools in communication systems. In this proposed scheme, it is used in order to encode and transmit the information. This technique can be implemented using the FWHT, which has the same complexity as the FFT used in OFDM, $N \log 2^N$, where N is the size of the Hadamard matrix.

The proposed systems decrease the effect of interference on the transmitted signals and lower the error probability in channels, which is verified through simulation results.

2. RELATED WORK

Dynamic exploration is going through to utilize the mmWave band for cell framework utilizing progressed equipment what's more, programming preparing power. Accordingly, proficient and exact channel assessment is one of the key necessities for the accomplishment of enormous mmWave MIMO frameworks [2]. Millimeter-wave (mmWave) correspondences and gigantic MIMO assume key parts in empowering gigabit remote access in straightaway age correspondences frameworks, dynamic examination region called mmWave gigantic MIMO [3]. The mmWave can address the test of data transmission lack for the up and coming age of remote cell correspondence frameworks. Then, at that point, the mmWave have a going from 30 GHz to 300 GHz [4]. The mmWave MIMO is a method for future 5G remote correspondence frameworks, the utilization of the mmWave recurrence band permits wide transfer speed for high information rates needed for future remote organizations. The execution useful for mmWave MIMO frameworks has more energy effective [5]. Presently, wideband mmWave channel assessment utilizing the half and half design, expecting to be an Orthogonal Recurrence Division Multiplexing (OFDM) framework and optimal settings, it is generally called supposed measure [6].

OFDM is broadly read for UWA channels zeroing in on intelligent recognition. Different strategies for cognizant location have been proposed which address single-transmitter or various transmitter framework, the benefits of high ghastly effectiveness and strength to multi-way bending of the remote channel [7]. OFDM is a well-known tweak plan, preparing and information transmission structure for OFDM transfer networks is suggested that empowers joint assessment of channel [8]. Most OFDM remote guidelines such as utilize pilot-based channel assessment methods for precisely disentangling the sent information bits [9].

Lately, MIMO-OFDM frameworks with enormous number of radio wires, alluded to as monstrous MIMO, the best in class Iterative Detection and Decoding (IDD) method is successful to further develop channel assessment nature of the MIMO-OFDM frameworks [10]. OFDM has been taken on in optical remote (OW) frameworks and MIMO strategy used to accomplish high information rate transmission, the presentation of indoor MIMO-OFDM Visible Light Communications (VLC) framework is upgraded by SVD-based versatile stacking, which is initially proposed in remote interchanges [11]. The mix of OFDM and MIMO transmission procedures are fascinating examination issue, like information location, channel assessment, etc [12].

3. METHODOLOGY

In wireless communication, SISO systems are used in communication for transmitting and receiving signals. In SISO system only one transmitting antenna and receiving antenna are used. In order to produce better results, additional transmitting and receiving antennas are required. In recent times mobile radio communication needs wide band transmission with peak spectral efficiency and mobility.

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MIMO technique has multiple transmitting and receiving antennas which increase the capacity of the channel; and spatial multiplexing makes the utilized frequency same as SISO systems. MIMO technique with OFDM is the best technique for achieving the efficiency and mobility of the system.

In MIMO OFDM systems, data attain the destination with different channel impairments like IAI, ISI and ICI. Interleaving makes the forward error correction and avoids the burst error. In reorder, the data and consecutive bytes of data are distributed. The major purpose of interleaving techniques is to increase the ability of error protection codes and orient the burst error.

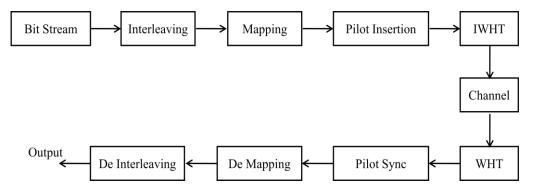


Figure 1 shows the block diagram of MIMO-OFDM system with interleaving WHT

A MIMO OFDM system with M_t transmit and M_r receive antennas are considered and a data bit is coded and interleaved. The coded bits are mapped into the data symbol depending on the type of modulation. In the proposed system, modulation is performed by 16 QAM and then the pilot symbol is inserted. Finally the output is fed into the FWHT.

Considering the random input symbol as, a $\{i_0, i_1, i_2, \dots, \}$ convolutional encoder consists of a set of shift registers each with a fixed delay. Every delay is not a negative integer multiple of a fixed integer. The data input is the nth subcarrier as a binary $m \times M_t$ vector $b_n = (b_0, \dots, b_{M_t-1})^T$. The binary data vector b_n is mapped into symbol vector S_n . The symbol from the input bit stream is fed into the shift register of the interleaver and the previous symbol in the register is the part of output signal. The structure of convolutional interleaver is shown below which has the shift registers and delayed signal values D(1), D(2), ..., D(N).

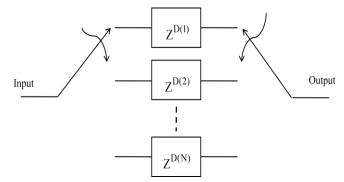


Figure 2 shows convolutional interleaver

ISSN: 0975-3583, 0976-2833 VOL 12, ISSUE 03, 2021

The interleaved symbols are represented by $Z^{-i_1}, Z^{-i_2}, \dots, Z^{i_N}$ The interleaved logical symbols are converted into the real integers and fed into the required modulation scheme. Thus the 16 QAM MIMO OFDM symbol is $\{x, x_1, x_3\}$. The symbol rate reduced after the modulation is given by

$$R_1 = \frac{R}{\log 2^N}$$

Where N is the constellation size. The serial to parallel converter converts the serial data into M number of parallel streams. Each stream constrains a small bandwidth of the spectrum. Thus the flat fading may appear. In the uniform cycle, Pilot symbol that is inserted may block the frequency bins. The design of pilot sequence must satisfy,

$$\Psi^{-m} \cdot \Psi^{-1} = \rho I_L$$

The optimal pilot design can be written as,

$$\Psi^{-m}.\Psi^{-1} = \begin{bmatrix} \Gamma_{0,0} & \Gamma_{0,0} & & \Gamma_{0,M-1} \\ \Gamma_{1,0} & \Gamma_{1,1} & & \Gamma_{1,M-1} \\ & & & \\ \Gamma_{M-1,0} & \Gamma_{M-1,0} & & \Gamma_{M-1,M-1} \end{bmatrix}$$

M is channel impulse response and $\Gamma \in \{1, 2, 3..., M-1\}$ is an L×L matrix

$$\Gamma_{K,i} = \left[A_k I_L\right]^H A_i F_L$$

In order to satisfy the condition

$$\Gamma_{K,i} = \begin{cases} \rho I_L & K = i \\ O_{L \times L} & K \neq i \end{cases} \quad i, K \in \{0, 1, 2, \dots M - 1\}, L \times L \text{ is a zero matrix }.$$

To estimate the optimal pilot sequence required within M_t channels of length L, the S pilot symbols split into R group. Assuming that the total power of the fixed symbol is ρ . Each group containing M_r pilots. That is S = R where $M_R > L$ and $R \ge M_T$ it makes

$$\Gamma_{K,K} = \rho I_L$$

To satisfy the condition $\Gamma_{K,i} = O_{L \times L}$ it is necessary that $\Gamma_{K,i}(c,d) = 0$ has to use the pilot structure. The τ^{th} pilot pattern of r^{th} group is

The angle terms are adjusted to decrease the PAPR. The sequence is assigned to all OFDM symbols to get the perfect sequence.

Thus the value of phase angle $\,\theta_{r,\tau}\,$ is fixed to zero. The optimal design of all the pilots is,

$$X_{v(r,\tau)}(m_{r,\tau}) = (\rho/S)$$

ISSN: 0975-3583, 0976-2833 VOL 12, ISSUE 03, 2021

3.1 FAST WALSH HADAMARD TRANSFORM

FWHT is a rectangular matrix with mutual orthogonal rows. All rows have a vertical vector which are associated with the values +1 and -1. The array $I(i_1, i_2, ..., i_n)$ and $J(j_1, j_2, ..., j_n)$ are expressed as,

$$I = \sum_{K=1}^{n} i_n 2^{n-1} = i_1 + i_2 \cdot 2 + i_3 \cdot 2^2 \dots i_n \cdot 2^{n-1}$$

And

$$J = \sum_{K=1}^{n} j_{n} 2^{n-1} = j_{1} + j_{2} \cdot 2 + j_{3} \cdot 2^{2} \dots j_{n} \cdot 2^{n-1}$$

Thus the Hadamard matrix is,

$$\begin{split} \mathbf{H}_{m} &= \left(-1\right)^{\mathbf{I}(1)\mathbf{J}(1) + \mathbf{I}(2)\mathbf{J}(2) \dots \dots \mathbf{I}(m)\mathbf{J}(m)} \\ & \mathbf{H}_{m} \otimes \mathbf{H}_{m} = m * \mathbf{I}_{m} \end{split}$$

Where I is the Identity matrix and H is Hadamard matrix. When the Hadamard transform is applied,

$$Y = H_m X \leftrightarrow H_m Y = m * X$$

If the matrix is multiplied with Hadamard matrix, it yields the same result as Fast Walsh Hadamard Transform (FWHT) on the reverse of the original matrix. Consider the output of mapper as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 \, \mathbf{x}_2 \end{bmatrix}^{\mathsf{T}}$$

Using $H_m \otimes H_m$ with the FWHT matrix, for m = 2.Y can be written as,

$$Y = H_2 X = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$$
$$H_2 X = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

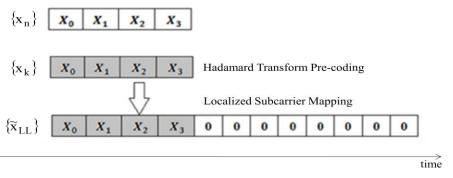
Compared to the Fast Fourier Transform (FFT), the Hadamard transform needs less space to store and is faster, along with a lesser complexity to calculate because it uses only real integer additions and subtractions, whereas the FFT requires complex values.

Hadamard OFDM transmit symbol for N=4, Q=3, M=12 is shown in Figure 3. The proposed method consists of MIMO OFDM with M_t transmit and M_r receive antennas. The bit sequence is independent of the transmitter input $(K) = [b_1, b_2, ..., b_k]^T$. The value of FWHT is fully made up of 1s and -1s WHT is defined as,

$$(H_m)_{(i+1)(j+1)} = (-1)^{(i-1)(j-1)}$$

Where I is the row and J is the column component of matrix to attain the signal M_t antennas.

ISSN: 0975-3583, 0976-2833 VOL 12, ISSUE 03, 2021



 $\widetilde{\mathbf{x}}_{\mathrm{LL}}$: Transmit Symbol for localized subcarrier mapping mode

Figure 3 An example of Hadamard OFDM transmit symbol for N=4, Q=3 and M=12

$$b_{ht(K)} = (H_m)_{(i, j)} \times b(K)$$

 $\begin{bmatrix} b_{ht(1,1)} & b_{ht(1,2)} & \dots & \dots & b_{ht(1,\frac{k}{m})} \\ b_{ht(2,1)} & b_{ht(2,2)} & \dots & \dots & b_{ht(2,\frac{k}{m})} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{ht(m,1)} & b_{ht(m,2)} & \dots & \dots & b_{ht(m,\frac{k}{m})} \end{bmatrix}$

The output of FWHT is mapped with the M_t antenna, and the signals are transmitted over $M_r \times M_t$ wireless channel by adding AWGN. Thus the transmitted signal is related to the received signal and is written as,

$$R_{j}(t) = \sum_{i=1}^{N_{t}} h_{i,j}(t) * X_{i}(t) + u_{j}(t) \qquad j = 0, 1, 2, ...M_{t}$$

Where $h_{i,j}(t)$ represents the channel impulse response between I transmitted antenna and J received antenna. MIMO form can be expressed as a matrix and it is given below

$$H(t) = \begin{bmatrix} h_{1,1}(t) & h_{1,2}(t) & h_{1,N_{t}}(t) \\ h_{2,1}(t) & h_{2,2}(t) & h_{2,N_{t}}(t) \\ h_{N_{r},1}(t) & h_{N_{r},2}(t) & h_{N_{r},N_{t}}(t) \end{bmatrix}$$

The encoding is done by Trellis coding. The source bits are Trellis encoded at rate 2/3 and 16 QAM mapping. Each QAM symbol is interleaved by π to achieve maximum diversity. When the cyclic prefix vector is added into the time domain signal, the resultant expression is

$$S_{CP_{i,j}}(i) = S_{j,M_c - M_g + 1}$$

Where $i = M_g + 1$, M_g is the Guard interval length. The *i*th sample time domain signal at the *j*th antenna

is

ISSN: 0975-3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$y_{i,j} = \sum_{i=0}^{M_t - 1} \sum_{l=0}^{M_g} R_{K,j,i}(i) S_{j,((i-l))_{NC}} + Z_{K,i}$$

Where $i = \sim M_C - 1, K = \sim M_r - 1, h_{K,j,l}(i)$ is the 1th channel tap gain between ith transmitted antenna and jth received antenna. AWGN noise is a noise with mean zero and σ^2 variance. Without loss of generality, the nth subcarrier of MIMO OFDM receiver is given as

$$Y_n = H_n^0 S_n + \sum_{d=1}^{N_c^{-1}} H_n^d S_{((n-d))_{N_c}} + Z_n$$

Where

$$\mathbf{Y}_{n} \equiv \left(\mathbf{Y}_{0,n} \dots \mathbf{Y}_{N_{r}-1,n}\right)^{T}$$
$$\mathbf{H}_{n}^{0} = \left\{\mathbf{H}_{K,j,n}^{0}\right\} \text{and} \mathbf{H}_{n}^{d} = \left\{\mathbf{H}_{K,j,n}^{d}\right\}$$

Which are both $M_r \times M_t$ channel matrices. Thus the received signal is

 $y = hb_nt + u$

4. RESULTS AND DISCUSSION

This section presents the simulation results of WHT with interleaving. The parameters taken into consideration are order of modulation and the total number of antennas used at the transmitter and receiver. The performances of FWHT with interleaving are compared under various modulation schemes and FWHT with ZF and STBC. The channel under consideration is Rayleigh fading channel and the receiver is assumed to have perfect channel state information.

Figure 4 shows the performance of interleaving (ILR), FWHT with interleaving (FWHT-ILR) using two transmitters and receivers and different modulation schemes. It indicates that a power gain of 6.5 dB is required to achieve an error rate of 10^{-3} for FWHT with interleaving for QPSK modulation and it is 7.7dB for FWHT without interleaving. This shows the effectiveness of FWHT with interleaving. Also the power gain that was achieved for 8 Phase Shift Keying (8-PSK) and 16 PSK are 8.5dB and 11.5 dB respectively for achieving the same error rate of 10^{-3} with interleaving WHT. This clearly indicates that QPSK modulation supports well for FWHT with interleaving.

Figure 5 shows the simulation performance of FWHT with interleaving for various numbers of antennas at the transmitter section. Since QPSK modulation performs well, it is preferred for modulation. Figure 5 indicates a power gain of 6.1dB, 6.3dB and 9.8dB that are required for achieving an error rate of 10⁻³. This clearly shows that the increase in the number of antennas at the transmitter section degrades the performance of bit error rate.

Figure 6 shows the BER performance of FWHT with interleaving using QPSK modulation, two antennas at the transmitter section and various number of antennas at the receiver section. It shows that the power gain is 10.5dB, 11,5dB, 13dB and 16.5dB respectively for eight, six, four and two number of receivers to get an error rate of 10^{-3} . It proves that with interleaving WHT, increase in number of receivers increases the performance of the system.

ISSN: 0975-3583, 0976-2833 VOL 12, ISSUE 03, 2021

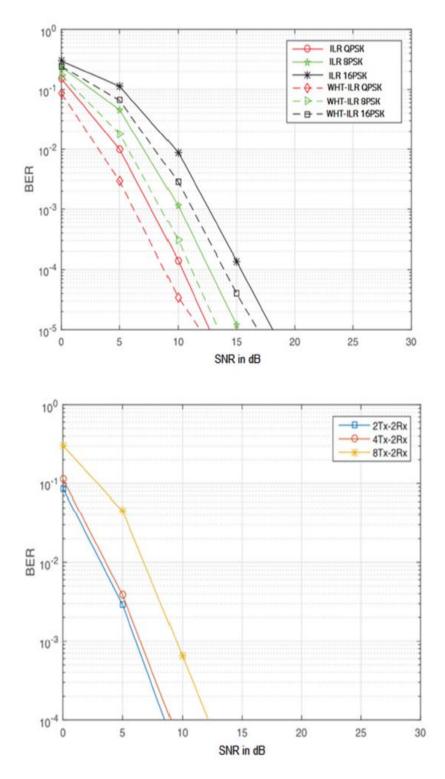


Figure 5 BER Comparison by increasing the number of transmit antennas



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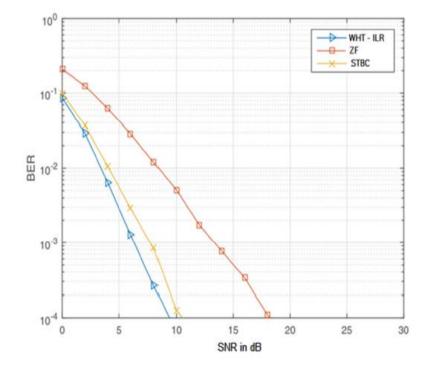


Figure 6 BER Comparison by increasing the number of receive antennas

Figure 7 BER Comparison with WHT – ILR, ZF and STBC

Figure 7 compares the performance of FWHT with interleaving against FWHT with ZF and WHT with STBC using QPSK modulation. It clearly shows that the effectiveness of performance is better for FWHT with interleaving than FWHT with ZF and STBC. From Figure 4 to Figure 7 it is clearly evident that WHT with interleaving works well with increase in number of receivers using QPSK modulation.

The proposed method decreases the occurrence of peak error compared to other OFDM system. The technique of FWHT is based upon the relationship between correlation property of OFDM input sequence and PAPR probability.

The average power of the input sequence represents the peak value of the autocorrelation. Hence the peak value of autocorrelation depends on the input sequence provided that the number of sub carriers remains unchanged. The FWHT is a non-sinusoidal, orthogonal linear transform and can be implemented by a butterfly structure as in FFT. This means that applying FWHT does not require an extensive increase of system complexity. FWHT decomposes a signal into a set of basic functions. These functions are Walsh functions, which are square waves with values of +1 or -1. The proposed Hadamard transform scheme may reduce the occurrence of the high peaks comparing the original OFDM system. It minimizes the auto correlation of an input sequence, and hence the PAPR problems are eliminated. Here the data generated is randomly interleaved and then modulated by 16 QAM technique and is evaluated. While comparing the data without interleaving to with interleaving, the latter one is more effective.

5. CONCLUSION

This chapter presented a different strategy with interleaving FWHT for MIMO-OFDM system using QPSK modulation. The proposed strategy enhances data reliability using interleaving and overcomes the limitations of WHT with ZF and FWHT with STBC. Simulation results have validated the performance of the proposed system.

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