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Banach's Banach-Metric Distance Hypothesis

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ABSTRACT:

Matthews introduced the idea of a partial metric area and obtained, among other results, Banak's contractive map of these distances. Later, S.J. O'Neill generalized Matthews' idea of partial scales, to establish links between these structures and topological aspects of field theory. Here, we get Banach's theory of the fixed point of full partial metric spaces in the sense of O'Neill. Therefore, Matthews fixed point theory remains a special case of our results. Keywords: Dualistic partial metric, partial metric, complete, quasi-metric, fixed point

INTRODUCTION:

Throughout this document, the letters R,R + , and N denote the real number group and the non-negative real number group and the natural number group, respectively. SG Matthews introduced the idea of partial metric space in [4] as part of a study of the symbolic implications of data flow networks. In particular, it established the exact relationship between partial metric spaces and the so-called balanced metric areas, and showed a partial metric generalization of the Banach contraction mapping theory. Remember that the partial measurement in a set (not empty) X is a function

$p: X\times X \to R^+$ such that for all x, y, z ∈X:

i. $x=y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$;

ii. $p(x, x) \leq p(x, y)$;

iii. $p(x, y) = p(y, x)$;

iv. $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

The partial metric area is a pair (X, p) where X is a non-blank set and a partial measurement in X. In [5], S.J. O'Neill proposed a significant change in Matthews' definition of partial scales, and was intended to extend his range from R + to R. In the following, partial scales will be called meaning O'Neill Partial bi-directional and pair scales (X, p) so that X is a non-blank set and the two-dimensional partial scale in X will be called the bi-directional partial measurement area. In this way, O'Neill developed several connections between partial scales and topological aspects of field theory. In addition, the pair (R, p), where p (x, y) = x∨y for all x, and ∈R provides a typical example of a double partial measurement area that does not form a partial metric area. Other interesting examples of partial binary (or partial) metric distances can be found from an arithmetic point of view in [1], [4], [6], [8], etc. Each binary metric fraction creates x x T0topology T (p) in X, which is based on the Open Balls family {Bp (x, ε): x∈ X, ε> 0}, where Bp (x, ε) = {y∈ X: p (x, y) 0. Immediately it follows that the (xn) sequence equals a two-dimensional subspace (X, p) converges to the point x∈X if and only if p (x, x) = limn → (p (x, xn). According to [5] (comparison [4]), the sequence (xn) n∈N in binary partial metric space (X, p) is called the Cauchy sequence when there is limn, m → ∞ p (xn, xm). The binary partial metric space (X, p) is said to be complete if all Cauchy The sequence (xn) n∈Nin X is converging, with respect to T (p),

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To the point xX where p (x, x) = limn, m → ∞ p (xn, xm). As noted earlier, motivated by applications in program validation, Matthews obtained [4] Banach's theory of the fixed point of full partial metric distances. Since partial (complete) partial scales provide a new way to generalize both the theoretical field approach and the metric approach to semantics (see [5], p. 314), it seems interesting to have a Benach theory of a fixed point in the range of double bias distances in this article. This kind of theory. In particular, Matthews' contraction mapping theory will be deduced as a special case from our results. 2. Banach’s fixed point theorem for complete dualistic partial metric spaces Before establishing our main result, we create some correspondence (mainly known) between binary partial scales and semi-metric areas. Our primary references for semi-metric areas are [2] and [3]. In our context by semi-metric in group X, we mean the real non-negative value function, the gift X x X, so that for all x, y, z ∈X:

i. d(x, y) = d(y, x) = 0 ⇔x=y,
ii. d(x, y)≤d(x, z) + d(z, y)

Aquasi-metric space is a pair (X, d) such that Xis a (nonempty)set and dis a quasi-metric on X.Each quasi-metric don Xgenerates a T0-topology T(d) on Xwhich has as a base the family of open d-balls {Bd(x, ε) : x∈X,ε > 0}, where Bd(x, ε) = {y∈X:d(x, y)< ε}for all x∈Xandε > 0.If dis a quasi-metric on X, then the function dsdefined on X×Xby ds(x, y) = max{d(x, y), d(y, x)}, is a metric on X. The proof of the following auxiliary results are analogous to theproofs of [4], Theorems 2.1 and 2.2 and [5], Definition 2.6 and Lemma2.7. However, we include such proofs in order to help to the reader. Lemma 2.1.If (X, p)is a dualistic partial metric space, then thefunction dp:X×X→R+defined by dp(x, y) = p(x, y)−p(x, x),is a quasi-metric on Xsuch that T(p) = T(dp).

Proof. Consider x, y ∈X. Then dp(x, y) = p(x, y)−p(x, x) is alwaysnonnegative because of p(x, x)≤p(x, y).
Now, we have to check that dpis actually a quasi-metric on X.
Let x, y, z ∈X. It is obvious that x=yprovides that dp(x, y) =

dp(y, x) = 0.Moreover, if dp(x, y) = dp(y, x) = 0 then p(x, y)−
p(x, x) = p(y, x)−p(y, y) = 0.Hence we obtain that x=y, since
p(x, y) = p(x, x) = p(y, y).Furthermore
dp(x, y) = p(x, y)−p(x, x)≤p(x, z) + p(z, y)−p(z, z)−p(x, x)=dp(x, z) + dp(z, y).
Finally we show that T(d) = T(dp).Indeed, let x∈Xand ε > 0and consider y∈Bdp(x, ε).Then dp(x, y) = p(x, y)−p(x, x)< εand, hence, p(x, y)< ε +p(x, x).Consequently y∈Bp(x, ε) andT(dp)⊆ T (d).Conversely if y∈Bp(x, ε) we have that p(x, y)< ε +p(x, x).Thus dp(x, y) = p(x, y)−p(x, x)< ε, y ∈Bdp(x, y)
andT(d)⊆ T (dp).

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Lemma 2.2. (compare [4], [5], [7]). A dualistic partial metric space(X, p)is complete if and only if the metric space (X, (dp)s)is com-plete. Furthermore limn→∞ (dp)s(a, xn) = 0 if and only if p(a, a) =limn→∞ p(a, xn) = limn,m→∞ p(xn, xm). Proof. First we show that every Cauchy sequence in (X, p) is aCauchy sequence in (X, (dp)s).To this end let (xn)nbe a Cauchysequence in (X, p).Then there exists α∈Rsuch that, given ε > 0,there is nε∈Nwith |p(xn, xm)−α|< ε for all n, m ≥nε.We conclude that (xn)nis a Cauchy sequence in (X, (dp)s).Next we prove that completeness of (X, (dp)s) implies complete-ness of (X, p).Indeed, if (xn)nis a Cauchy sequence in (X, p) thenit is also a Cauchy sequence in (X, (dp)s).Since the metric space(X, (dp)s) is complete we deduce that there exists y∈Xsuch thatlimn→∞(dp)s(y, xn) = 0.By (2.1) we follow that (xn)nis a convergentsequence in (X, p).Next we prove that limn,m→∞ p(xn, xm) = p(y, y).Since (xn)nis a Cauchy sequence in (X, p) it is sufficient to seethat limn→∞ p(xn, xn) = p(y, y).Let ε > 0 then there exists n0∈Nsuch that (dp)s(y, xn)< ε, whenever n≥n0.This shows that (X, p) is complete.Now we prove that every Cauchy sequence (xn)nin (X, (dp)s) isa Cauchy sequence in (X, p).Let ε=1/2.Then there exists n0∈Nsuch that dp(xn, xm) dp(xn, xn0) + p(xn, xn) = dp(xn0, xn) + p(xn0, xn0),then|p(xn, xn)|=|dp(xn0, xn) + p(xn0, xn0)−dp(xn, xn0)|≤dp(xn0, xn) + |p(xn0, xn0)|+dp(xn, xn0)≤2(dp)s(xn, xn0) + |p(xn0, xn0)| 0,thereexists nε∈Nsuch that (dp)s(xn, xm)< ε because ofp(xn, xn) = dp(xm, xn) + p(xm, xm)−dp(xn, xm).Therefore limn→∞ p(xn, xn) = a.On the other hand, |p(xn, xm)−a|=|p(xn, xm)−p(xn, xn) + p(xn, xn)−a|≤dp(xn, xm) + |p(xn, xn)−a|< εfor all n, m ≥nε.Hence limn,m→∞ p(xn, xm) = aand (xn)nis aCauchy sequence in (X, p).We shall have established the lemma if we prove that (X, (dp)s) iscomplete if so is (X, p). Let (xn)nbe a Cauchy sequence in (X, (dp)s).Then (xn)nis a Cauchy seuqence in (X, p), and so it is convergentto a point y∈Xwithlimn,m→∞ p(xn, xm) = limn→∞ p(y, xn) = p(y, y).Then, given ε > 0,there exists nε∈Nsuch that p(y, xn)−p(y, y)< ε and p(y, y)−p(xn, xn)< εwhenever n≥nε.As a consequence we havedp(y, xn) = p(y, xn)−p(y, y)< ε,

CONCLUSION:

In light of the previous natural result, one may ask whether the contractionary condition (1) can be replaced in our statement of theory with the corresponding contraction condition (2) above. The following simple example shows that this is not the case.

REFERENCES:

[1] M.H. Escardo,Pcf extended with real numbers, Theoretical ComputerScience 162 (1996), 79–115.

[2] P. Fletcher and W.F. Lindgren,Quasi-uniform spaces, MarcelDekker, New York, 1982

[3] H.P.A. K¨unzi, Handbook of the History of General Topology, vol. 3,ch. Nonsymmetric distances and their associated topologies: About theorigins of basic ideas in the area of asymmetric topology, pp. 853–968,Kluwer Acad. Publ., 2001, pp. 853–968.

[4] S.G. Matthews,Partial metric topology, Proc. 8th Summer Confer-ence on General Topology and Applications, Ann. New York Acad. Sci.,vol. 728, 1994, pp. 183–197.

[5] S.J. O'Neill,Partial metrics, valuations and domain theory, Proc.11th Summer Conference on General Topology and Applications, Ann.New York Acad. Sci, vol. 806, 1996, pp. 304–315.

[6] S. Romaguera and M. Schellekens,Quasi-metric properties ofcomplexity spaces, Topology Appl. 98 (1999), 311–322.

[7] S. Oltra, E.A. S. Romaguera and S´anchezP´erez,Bicompletingweightable quasi-metric spaces and partial metric spaces, Rend. CircoloMat. Palermo 51 (2002), 151–162.

[8] M. Schellekens,The smyth completion: a common foundation fordenonational semantics and complexity analysis, Proc. MFPS 11, Elec-tronic Notes in Theoretical Computer Science, vol. 1, 1995, pp. 211– 232.