

# A Case Study on the Use of Soft Sets in Decision-Making

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## INTRODUCTION

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. But many complicated problems in economics, engineering, environment, social science, medical science, etc., involve data which are not always all crisp. We cannot always use the classical methods because of various types of uncertainties present in these problems. The important existing theories viz. theory of probability, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2,3], theory of vague sets [4], theory of interval mathematics [3,5], theory of rough sets [6] can be considered as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties as pointed out in [7]. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theories; and consequently, Molodtsov [7] initiated the concept of soft theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work [7]. Soft sets are called (binary, basic, elementary) neighbourhood systems [8] and are a special case of context dependent fuzzy sets, as defined by Thielle [9]. In [10], we have made a theoretical study of the "soft set theory". In this paper, we present an application of soft sets in a decision making problem with the help of rough mathematics of Pawlak [11]. Earlier, a rough set representation, and hence, application done by Lin [12] and Yao [8]. We have used here an almost analogous representation of the soft sets in the form of a binary information table. 2.

## PRELIMINARIES

In this section, we present the notion of soft sets introduced by Molodtsov in [7], and some useful definitions from [11] on rough mathematics. 2.1. Definition of Soft Set Let  $W$  be an initial universe set and let  $E$  be a set of parameters.

## DEFINITION

2.1. (See [7].) A pair  $(F, E)$  is called a soft set over  $U$  if and only if  $F$  is a mapping of  $E$  into the set of all subsets of the set  $U$ : i.e.  $F : E \rightarrow P(U)$ , where  $P(U)$  is the power set of  $U$ . In other words, the soft set is a parameterized family of subsets of the set  $U$ . Every set  $F(e)$ , for  $e \in E$ , from this family may be considered as the set of elements of the soft set  $(F, E)$ , or as the set of approximate elements of the soft set. As an illustration, let us consider the following examples (quoted from [7]). (1) A soft set  $(F, E)$  describes the attractiveness of the houses which Ivlr.  $X$  is going to buy.  $U$  = the set of houses under consideration.  $E$  = the set of parameters. Each parameter is a word or a sentence.  $E = \{\text{expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair}\}$ . In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. It is worth noting that the sets  $F(e)$  may be empty for some  $e \in E$ .

(2) Zadeh's fuzzy set may be considered as a special case of the soft set. Let  $A$  be a fuzzy set of  $U$  with membership  $\mu_A$ , i.e.,  $\mu_A$  is a mapping of  $U$  into  $(0, 1]$ . Let us consider the family of  $\alpha$ -level sets for the function  $\mu_A$  given by  $F(\alpha) = \{x \in U : \mu_A(x) \geq \alpha\}$ ,  $\alpha \in [0, 1]$ . If we know the family  $F$ , we can find the

functions  $PA(x)$  by means of the following formulae:  $PA(x) = \dots$  Thus, every Zadeh's fuzzy set  $A$  may be considered as the soft set  $(I; [0, 1])$ . (3) Let  $(X, \tau)$  be a topological space, that is,  $X$  is a set and  $\tau$  is a topology, in other words,  $\tau$  is a family of subsets of  $X$ , called the open sets of  $X$ . Then, the family of open neighbourhoods  $T(x)$  of point  $x$ , where  $T(x) = \{V \in \tau : x \in V\}$ , may be considered as the soft set  $(T(x), \tau)$ .

The way of setting (or describing) any object in the soft set theory principally differs from the way in which we use classical mathematics. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated and we cannot find the exact solution. So, in the second step, we introduce the notion of approximate solution and calculate that solution. In the soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer: with the help of words and sentences, real numbers, functions, mappings, and so on. It means that the problem of setting the membership function or any similar problem does not arise in the soft set theory.

#### DEFINITION

A knowledge representation system can be formulated as follows: knowledge representation system is a pair  $S = (U, A)$ , where  $U =$  a nonempty, finite set called the universe.  $A =$  a nonempty, finite set of primitive attributes. Every primitive attribute  $a \in A$  is a total function  $a : U \rightarrow V$ , where  $V$ , is the set of values of  $a$ , called the domain of  $a$ . DEFINITION 2.3. (See [11].) With every subset of attributes  $B \subseteq A$ , we associate a binary relation  $IND(B)$ , called an indiscernibility relation, defined by

$$IND(B) = \{(x, y) \in U^2 : \text{for every } a \in B, a(x) = a(y)\}.$$

Obviously,  $IND(B)$  is an equivalence relation and

$$IND(B) = \bigcap_{a \in B} IND(a).$$

Every subset  $B \subseteq A$  will be called an attribute. If  $B$  is a single element set, then  $B$  is called primitive, otherwise the attribute is said to be compound. Attribute  $B$  may be considered as a name of the relation  $IND(B)$ , or in other words-as a name of knowledge represented by an equivalence relation  $IND(B)$ .

#### DEFINITION

2.4. (See [11].) Let  $R$  be a family of equivalence relations and let  $A \in R$ . We will say that  $A$  is dispensable in  $R$  if  $IND(R) = IND(R - \{A\})$ ; otherwise  $A$  is indispensable in  $R$ . The family  $R$  is independent if each  $A \in R$  is indispensable in  $R$ ; otherwise  $R$  is dependent. If  $R$  is independent and  $P \subseteq R$ , then  $P$  is also independent.  $Q \subseteq P$  is a reduct of  $P$  if  $Q$  is independent and  $IND(Q) = IND(P)$ . Intuitively, a reduct of knowledge is its essential part, which suffices to define all basic concepts occurring in the considered knowledge, whereas the core is in a certain sense its most important part. The set of all indispensable relations in  $P$  will be called the core of  $P$ , and will be denoted  $CORE(P)$ . Clearly,  $CORE(P) = \bigcap_{Q \in RED(P)} Q$ , where  $RED(P)$  is the family of all reducts of  $P$ . The use of the concept of the core is twofold. First, it can be used as a basis for computation of all reducts, for the core is included in

every reduct, and its computation is straightforward. Secondly? the core can be interpreted as the set of the most characteristic part of knowledge, which cannot be eliminated when reducing the knowledge.

AN APPLICATION OF SOFT SET THEORY Molodtsov [7] presented some applications of the soft set theory in several directions viz. study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. In this section, we present an application of soft set theory in a decision making problem with the help of rough approach [11]. The problem we consider is as below. Let  $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ , be a set of six houses,  $E = \{\text{expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair}\}$  be a set of parameters.

Consider the soft set  $(F, E)$  which describes the 'attractiveness of the houses', given by  $(F, E) = \{\text{expensive houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{beautiful houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{wooden houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{modern houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{in bad repair houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{cheap houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{in good repair houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{in the green surroundings houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}\}$ . Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters 'beautiful', 'wooden', 'cheap', 'in the green surroundings', 'in good repair', etc., which constitute the subset  $P = \{\text{beautiful; wooden; cheap; in the green surroundings; in good repair}\}$  of the set  $E$ . That means, out of available houses in  $U$ , he is to select that house which qualifies with all (or with maximum number of) parameters of the soft set  $P$ . Suppose that, another customer nfr. Y wants to buy a house on the basis of the sets of choice parameters  $Q \subset E$ , where,  $Q = \{\text{expensive; beautiful; in the green surroundings; in good repair}\}$ , and klr. Z wants to buy a house on the basis of another set of parameters  $R \subset E$ . The problem is to select the house which is most suitable with the choice parameters of Mr. X. The house which is most suitable for Mr. X, need not be most suitable for Mr. Y or Mr. Z as the selection is dependent upon the set of choice parameters of each buyer. To solve the problem, we do some theoretical characterizations of the soft set theory of Molodtsov, which we present below.

## CONCLUSION

The soft set theory of Molodtsov [7] offers a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. Molodtsov in [7] has given several possible applications of soft set theory. In the present paper, we give an application of soft set theory in a decision making problem by the rough technique of Pawlak [11]

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