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CALCULATION OF X-RAY LASER INVERSION IN THE GROUND STATE OF OXYGEN

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ABSTRACT
The recent availability and progress of intense ultra short pulse laser systems has led to growing interest in the development of high-repetition-rate soft x-ray lasers driven by short laser pulses. With the help of these intense laser pulses any kind of matter can be rapidly ionised on a time scale comparable to or shorter than the characteristic relaxation times of ion states. This can result in a transient population inversion between the ground and excited ion states and amplification of radiation in the soft-x-ray region.

The purpose of this paper entitled "Calculation of X-ray lasers inversion in the ground state of oxygen" is to calculate some laser systems so as to understand lasing transitions and obtain gain in some suitable media in the range of X-ray wavelengths.

INTRODUCTION: The development of lasers towards the shorter wavelength region and the possibility of their applications in different branches of science and industries in a more fruitful way have led the researchers to build up new lasers in the range of X-ray wavelengths. Many theoretical investigations have been proposed and a lot of experimental demonstrations have been carried out since the early 1970's to produce a successful X-ray laser. A number of laser systems such as collisional excitation, recombination, photoionization, capillary discharge, chirped pulse amplification (CPA) etc. have been investigated and in some cases small gain in the X-ray laser (XRL) have been experimentally demonstrated. Still, we have to wait to obtain a practical X-ray laser owing to a number of practical difficulties.

X-ray lasers using laser plasmas as active medium have been extensively reported in literatures following both the electron collisional and recombination schemes[1,2]. Various workers have demonstrated the collisional pumped x-ray lasers in the neon isoelectronic sequences of 3p-3s transitions in laser plasmas[3,4]. One of the most popular soft x-ray laser approaches is based on the optical field ionization of atoms in an intense laser field[5,7]. This ionization mechanism is very selective and allows one to strip all atoms to a particular ionization stage. In the case of a linearly polarised laser field the free electron can be left relatively could which allows one to create a transient population inversion between excited and ground ion states during the three-body recombination cascade. Uptil now, theoretical studies of a possible x-ray lasing to the ground state ions were concentrated on H-like and Li-like ions[5,6]. Experimental results[8-10] have demonstrated evidence of amplification in H-like Li ions at $135\overset{0}{A}$ on the L_α transition. This evidence is not overwhelming and there are still some contradictions[11,12]. In this paper I try to identify more suitable routes and scheme for the creation of population inversion and soft x-ray lasing to the ground state for OIII ions. Saha's ionization formula has been used to analyse the time evolution of fractional population inversion and gain.

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THEORY:

1.1 GROUND STATE LASERS AND ENERGY LEVEL SCHEME:

Ground-state lasers are nothing unusual. For example, of the 6940 $\overset{0}{A}$ Ruby laser is the first laser ever to be operated. Ground state optical maser is known to be operated by higher pumping rates as compare to the lasers working with transitions between excited levels and they can be well modelled by a simple three level scheme. Therefore, to find an optimal way for the creation of an inversion between the ground and excited states of an ion we also consider an idealized three level model as shown in the Fig. 1.1.

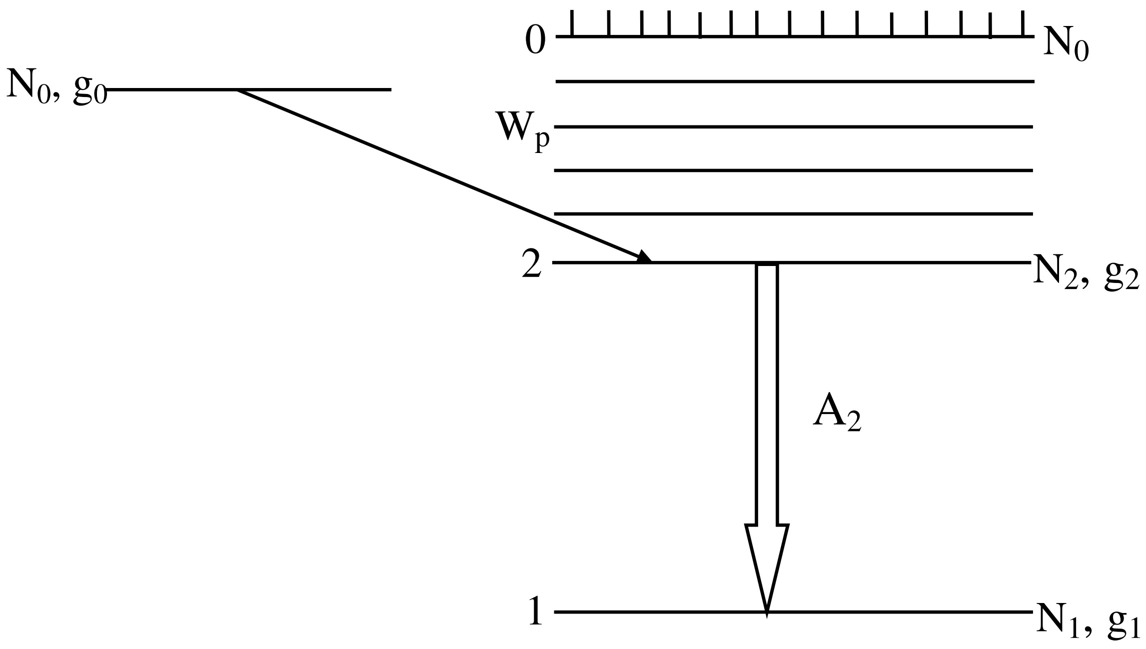

Fig. 1.1. Three level scheme for ground state soft X-ray laser

Two neighbour ion stages connected by a pumping process having the rate $W_p(S^{-1})$ are assumed. This pumping produces the required ion population directly in the upper laser state. Here A_{21} is the probability of the spontaneous radioactive decay.

1.2. THE GENERAL CONDITIONS FOR POPULATION INVERSION TO THE GROUND STATE:

The basic theory is concerned with the three-body recombination cascade process. The rate equations for populations of the various levels then written in the form for populations N_i of the various states are

$$\left.\begin{aligned} \dot{N}_0 &= -N_0 W_p \\ \dot{N}_2 &= N_0 W_p - N_2 A_{21} \end{aligned}\right\} \qquad \text{........1.1}$$

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$$\dot{N}_1 = N_2 A_{21}$$

Where $N = N_0 + N_1 + N_2$ = constant population at the initial conditions are $N_0 = N - n$, $N_2 = 0$, $N_1 = n$ with 'n' being the initial population of the ground state. The formal solution of the above rate equations with W_p depending on time is given by,

$$N_0 = (N-n)\exp\left[-\int_0^t W_p(t')dt'\right]$$

$$N_2 = (N-n)\exp\left[-A_{21}t\int_0^t W_p(t')dt'\right]\times\exp\left[-\int_0^{t'}\{\ W_p(t'') - A_{21}\ \}dt''\right]dt'$$

..... 1.2

The corresponding time evolution of the population

$$\Delta N = N_2 - N_1\left(\frac{g_2}{g_1}\right) \qquad \text{is}$$

$$\Delta N = \left(\frac{g_2}{g_1}\right)\Bigg\{(N-n)\exp(-A_{21}t)\int_0^t\left[W_p(t')\frac{g_2}{g_1} + A_{21}\right]$$

$$\times\exp\left[-\int_0^t\{W_p(t'') - A_{21}\}dt''\right]dt' + (N-n)\exp(-A_{21}t) - N\Bigg\} \quad \text{.... 1.3}$$

The general consideration we can assume that the pumping rate is constant during the short interval of inversion. For this case

$$\Delta N = \frac{g_2}{g_1}\Bigg\{(N-n)\exp(-A_{21}t\int_0^t\left[\left\{W_p(t')\frac{g_1}{g_2} + A_{21}\right\}\times\exp\{-(W_pt' - A_{21}t')\}\right]dt'$$

$$+ (N-n)\exp(-A_{21}t) - N\Bigg\}$$

$$= \frac{g_2}{g_1}\Bigg\{(N-n)\exp(-A_{21}t)\left[\frac{g_1}{g_2}W_pt + A_{21}t\right]\times\exp\{-(W_pt - A_{21}t\}$$

$$+ (N-n)\exp(-A_{21}t) - N\Bigg\}$$

$$= \frac{g_2}{g_1}\Bigg\{(N-n)\exp(-A_{21}t)\left[\left(\frac{g_1}{g_2}\frac{W_p}{A_{21}} + 1\right)A_{21}t\right]\times\exp\left[-\left(\frac{W_p}{A_{21}} - 1\right)A_{21}t\right]$$

$$+ (N-n)\exp(-A_{21}t) - N\Bigg\}$$

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$$= \frac{g_2}{g_1} N\left(1-\frac{n}{N}\right)\left\{\exp(-A_{21}t)\left[\left(\frac{g_1}{g_2}\frac{W_p}{A_{21}}+1\right)A_{21}t\right]\right.$$

$$\left.\times\exp\left[-\left(\frac{W_p}{A_{21}}-1\right)A_{21}t\right]+\exp(A_{21}t)-\frac{1}{1-n/N}\right\} \qquad \text{..... 1.4}$$

Now putting

$$\beta = n/N, \qquad \gamma = A_{21}/W_p, \qquad \tau = A_{21}t$$

$$\frac{\Delta N}{N} = \frac{g_2}{g_1}(1-\beta)\left\{\exp(-\tau)\left(\frac{g_1}{g_2}+\gamma\right)\frac{\tau}{\gamma}\times\exp\left[-\left(\frac{1}{\gamma}-1\right)\tau\right]\right.$$

$$\left.+\exp(-\tau)-\frac{1}{1-\beta}\right\}$$

$$= \frac{g_2}{g_1}(1-\beta)\left\{\frac{\tau}{\gamma}\exp(-\tau)\left(\frac{g_1}{g_2}+\gamma\right)+\exp(-\tau)-\frac{1}{1-\beta}\right\} \qquad \text{.....1.5}$$

For special case $\gamma = 1$, the above equation becomes

$$\frac{\Delta N}{N} = \frac{g_2}{g_1}(1-\beta)\left\{\exp(-\tau)\left(\frac{g_1}{g_2}+1\right)\tau\times\exp(-\tau)-\frac{1}{1-\beta}\right\} \qquad \text{......1.6}$$

1.3 **FRACTION OF ATOMS IONIZED (SAHA'S EQUATION)**

Saha ionization equation[14,15,16 which] represents the relation between fraction atoms ionized, the pressure, temperature and the ionization energy, which is written as

$$\frac{n_e n_i}{n_a}\left(\frac{M_e KT}{2\pi\hbar^3}\right)^{\frac{2}{3}}\frac{2g^+}{g_0}\exp\left(-\frac{Q}{RT}\right) \qquad \text{....1.7}$$

Where n_e is the number of electrons per unit volume, n_i is the number of ions, n_a is the number of neutral atom per unit volume, M_e is the electronic mass, g_0 and g^+ represents respectively the degeneracies of the neutral atoms and that of the ionized atom, and Q is the binding energy of the first electron. The formula is applied to a simple reaction like

$$Ca = Ca^+ + e - U \qquad \text{.......1.8}$$

The logarithmic form of equation 4.7 is written as

$$\log\left(\frac{f^2}{1-f^2}\right)P = -\frac{U}{2.3RT}+2.5\log T-6.5 \qquad \text{.....1.9}$$

Where f is the fraction of atoms ionized, P is the total pressure, U is the heat of ionization and R is the gas constant. Equation 1.9 shows that the fraction of atom ionized depends on U, P, and T. Further at a certain temperature the fraction of ionized atom depends on pressure only. Using equation 1.9 one can calculate the fraction (%) ionized at various values of temperature and pressure figure 1-2 is a rough schematic of the results obtained by Saha[13] for calcium. The striking fact is that when T is held constant, ionization increases as pressure decreases. In other words, one expects more at the top of the chromospheres than at the bottom. (For

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example, one can take the temperature at the photosphere as 7500^0K and the pressure there as 1 atmosphere (considered a reasonable value) then for calcium f = 0.34 i.e. 34% of calcium atoms would be ionized). It shows that calcium begins to get ionized at temperature of about 4000^0K and ionization is promoted by a reduction in pressure. But actually electrons arise from many other sources and therefore it is better to take its concentration as an independent constituent.

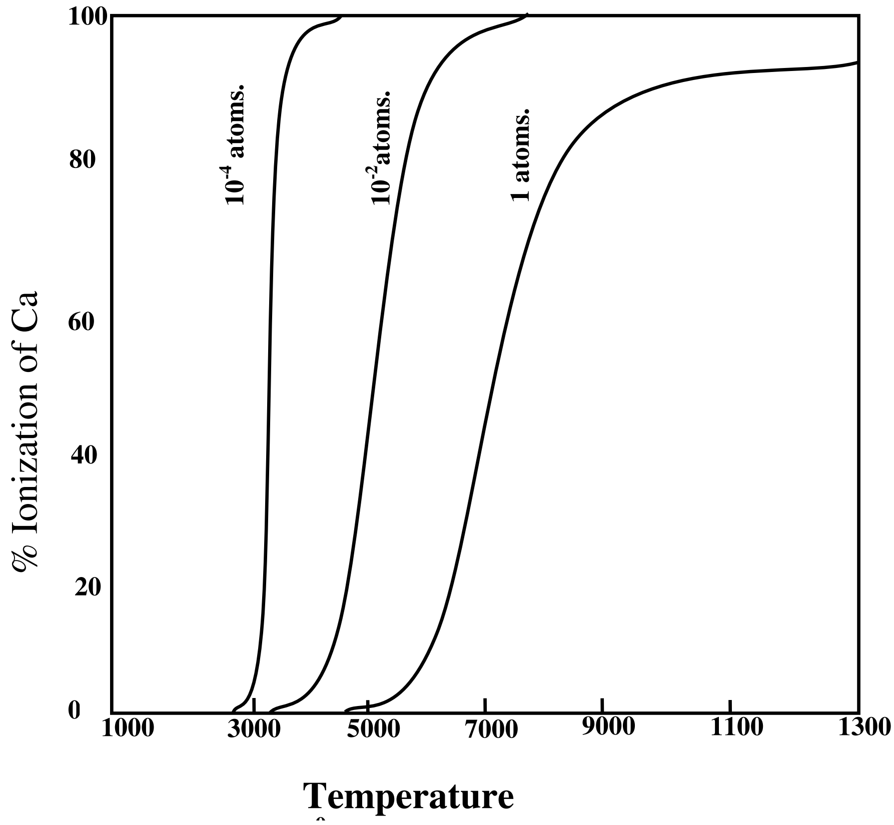

Fig. 1-2. Ionization of Ca at various temperatures at 1 atoms, 10^{-2} atoms, 10^{-4} atoms. Pressures, according to the Saha ionization equation 4.9.

1.4 COMPARISION OF FRACTION OF ATOMS IONIZED AND FRACTION OF INVERSION:

The fraction of inversion can be simplified from equation (1.5) as

$$\frac{\Delta N}{N}\frac{g_1}{g_2}\frac{1}{1-\beta} = \exp(-\tau) + \left(1+\frac{g_1}{g_2}\right)\tau\exp(-\tau) - \frac{1}{1-\beta}$$

$$\Rightarrow x\frac{g_1}{g_2}\frac{1}{1-\beta} + \frac{1}{1-\beta} = \exp(-\tau) + \left[1+\left(1+\frac{g_1}{g_2}\right)\tau\right]$$

Where $x = \frac{\Delta N}{N}$ is the fraction of inversion.

Taking the logarithm, we have

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$$\log\left(x\frac{g_1}{g_2}\frac{1}{1-\beta}+\frac{1}{1-\beta}\right)=-\tau+\log\left[1+\left(1+\frac{g_1}{g_2}\right)\tau\right]$$

$$\Rightarrow \log\left\{\left(x\frac{g_1}{g_2}+1\right)\frac{1}{1-\beta}\right\}=-\tau+\log\left[1+\left(1+\frac{g_1}{g_2}\right)\tau\right]$$

$$\Rightarrow \log\frac{1}{1-\beta}+\log\left(x\frac{g_1}{g_2}+1\right)=-\tau+\log\left[1+\left(1+\frac{g_1}{g_2}\right)\tau\right]$$

$$\Rightarrow \log\left(x\frac{g_1}{g_2}+1\right)=-\tau+\log\left[1+\left(1+\frac{g_1}{g_2}\right)\tau\right]-\log\frac{1}{1-\beta} \qquad ......1.10$$

The equation (1.7) and (1.10) shows similarity in both L.H.S. and R.H.S. The graphs shows in the Fig.1-2 describe the variation of percentage of ionization of Ca at various temperatures at 1 atoms, 10^{-2} atoms and 10^{-4} atoms according to equation (1.7). Fig.1-3 shows the fraction of population inversion of OIII for different values of β (β = 0.01, 0.03, and 0.05). These two graphs have been plotted only to indicate the analogies between Saha equation of thermal ionization and equation (1.10). The Curves indicate that the phenomena of population inversion and ionization (thermal) are analogous.

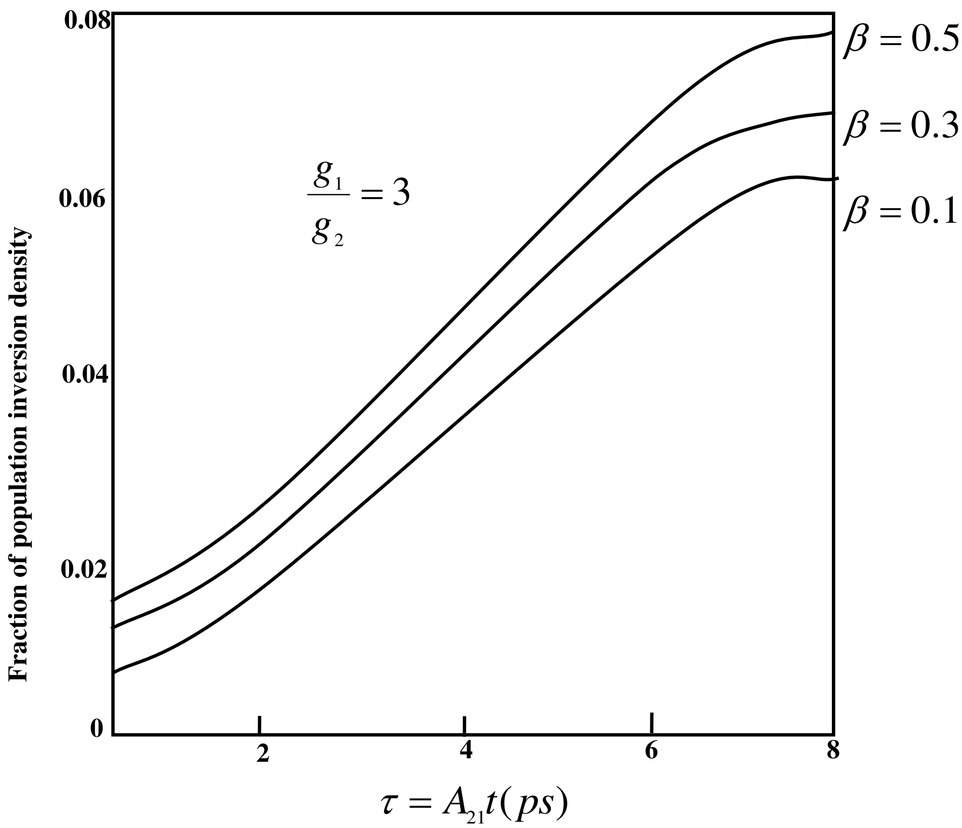

Fig. 1-3. Fraction of population inversion density Vs time in (ps)

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1.5 OIII ENERGY LEVELS AND RELATIVE POPULATION INVERSION:
I turn to the discussion of ion candidates that are more favourable for the creation of an inversion to the ground state. We consider low-charged ions with a partially filled L shell. In Fig. 1-4 this is applicable specifically for transitions in OIII ions. If the intensity of laser radiation is high enough to ionize most of the oxygen atoms to the next ionization stage OIV and if the electrons are left cold then due to the three body recombination cascade on inversion can be created on 2p3s ^{3}P - 2p^2 ^{3}P transition in OIII ions[17]. The gain coefficient G for a Doppler broadened Laser transition dominant in the high temperature plasma is expressed as

$$G = \frac{\lambda^3}{8\pi} A_{ul} \left(\frac{M}{2\pi kT} \right)^{1/2} g_u \left\{ \frac{N_u}{g_u} - \frac{N_2}{g_1} \right\} \text{ cm}^{-1} \qquad \ldots\ldots(1.11)$$

Expressing N_j in terms of electron and ion densities n_e and $n_i = n_e / \overline{Z}$ of a plasma formed from a target element of atomic number Z_a as

$$N_j = \left(\frac{n_j}{n_l} \right) \left(\frac{n_l}{n_i} \right) \left(\frac{n_i}{n_e} \right) n_e = \left(\frac{n_j}{n_l} \right) \left(\frac{\beta}{\overline{\overline{Z}}} \right) n_e$$

Where n_j is the number density of OIII ions in all levels. B = (n_l/n_i) is the fraction of OIII ions, n_i is the total ion density, $\overline{Z}$ is the average ionic charge state where $\overline{Z} = \left(\frac{2}{3} \right) Z_a T_{ea}^{\ 1/3}$, Substituting the values in equation (1.11).

We have

$$G(cm^{-1}) = \frac{1.74 \times 10^6 M^{1/2}}{T_i^{1/2}} A_{ul} g_u \left(\frac{N_u'}{g_u} - \frac{N_l'}{g_l} \right) \left(\frac{\beta}{\overline{\overline{Z}}} \right) n_e$$

$$= \frac{1.74 \times 10^6 \lambda^3 M^{1/2}}{T_i^{1/2}} A_{ul} N_u' \left(1 - \frac{N_l'}{N_u'} \frac{g_u}{g_l} \right) \left(\frac{\beta}{\overline{\overline{Z}}} \right) n_e \qquad \ldots\ldots.(1.12)$$

The quantity in the bracket is always less than unity for positive gain. So for our calculation an arbitrary value of .03 has been chosen for this quantity.

RESULTS AND DISCUSSION
In Fig. 1.5 (a, b, c) the time dependence of the relative population inversion or the fraction of atoms undergoing inversion are shown for three different ratios of g_1/g_2 and the initial populations of the ground levels.

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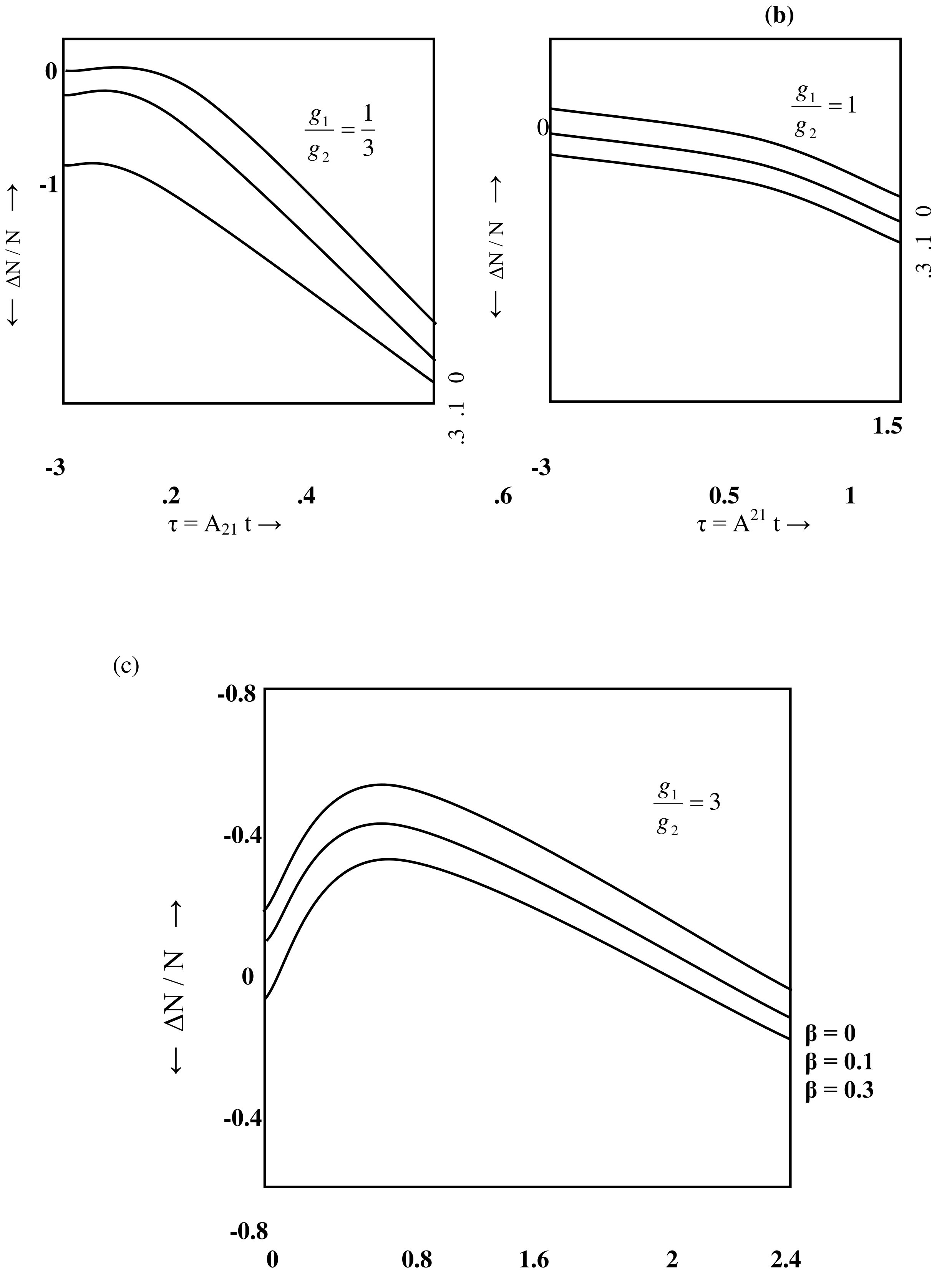

Fig. 1.5: Time dependence (a), (b) & (c) of relative population inversion for different ratios and initial populations of the ground state for $\gamma = A_{21} / W_p = 1$

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Table-1: Fractional population inversion different values of statistical weight, spontaneous decay constant and the initial population of the ground state with $A_{21} / W_p = 1$ 0f OIII transitions.

g_1/g_2	$\beta = n/N$			g_1/g_2	$\beta = n/N$			g_1/g_2	$\beta = n/N$		
	0	0.1	0.3		0	0.1	0.3		0	0.1	0.3
	0	-0.332	-0.931		0	-0.112	-0.340		-o.662	-0.331	-0.132
	0.734	-0.215	-0.874		-0.823	-0.393	-0.542	3	0.073	0.172	0.12
1/3	-0.172	-0.692	-1.056	1	-0.965	-0.855	-0.981		0.235	0.221	0.252
	-0.551	-1.018	-1.345		-0.629	-0.974	-0.652		0.289	0.181	0.02
	-1.025	-1.466	-1.931		-0.516	-0.521	-0.721		0.243	0.112	0.002
	-2.052	-1.701	-2.123		-0.453	-0.289	-0.201		0.003	0.022	0.002
	-2.511	-1.894	-2.325		-0.323	-0.223	-0.192		0.002	-0.032	0.001

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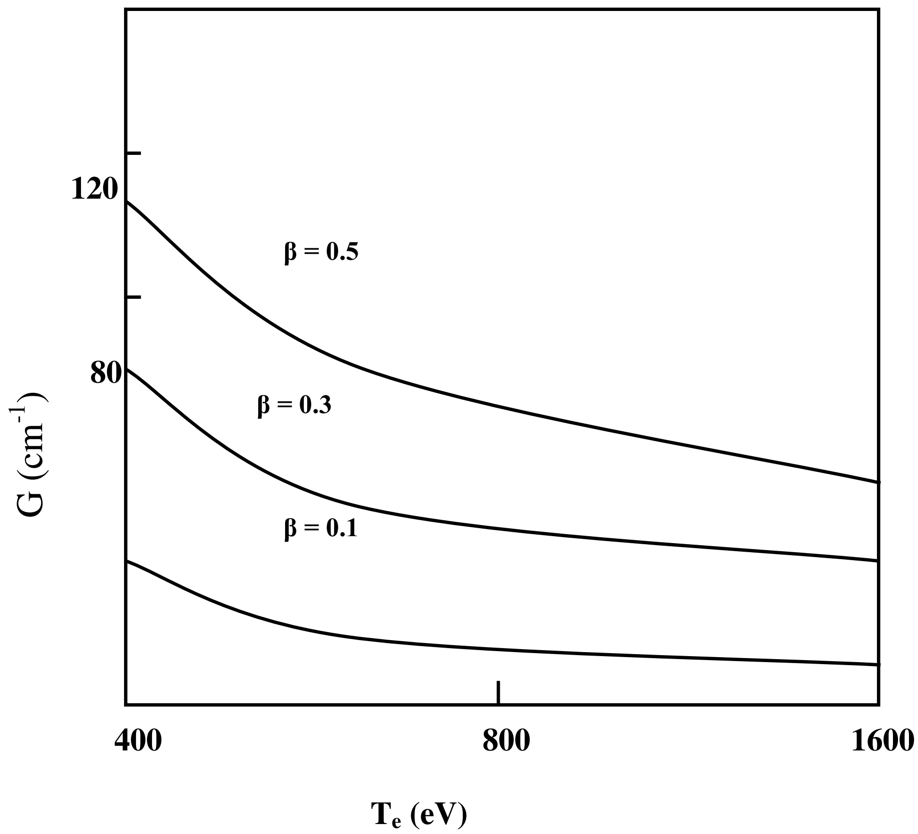

Fig. 1.6: Gain co-efficient for different electron temperature (eV).

These curves indicate how the ratio controls the time evaluation of the population inversion. For the ratio =1 the relative population inversion always assumes negative value. Significant relative population inversion appears only for values of the ratio greater than unity [in Fig.1.5(c)]. The parameter also assumes significance in the time evaluation of the relative population inversion. We have seen Fig. 1.5(c) that for =1, that is for high pumping rate ($A_{21} = W_p$), $\beta = 0$ and $\tau = 0.8$ (arbitrary value) the fraction of inversion is maximum, indicating gain. Fig.1.6 indicates the variation of gain for different ion temperatures. It is worth while to note that the analysis given here is applicable to OIII ions but it is applicable in general to three level ground state lasers. We have indicated the possible use of the ionization formula since x-ray laser usually involves ionized state, Saha's equation should open up a new avenue for understanding the process of short wave length lasing.

REFERENCES:

1. Keanes, C.T.; Ceglio, N.M.; Mac Gowan, B.J.; Mathews, D.L.; Nilson, D.G.; Trebes, J.E. and Whelean, D.A.; J. Phys.; B22, 3343, (1989).
2. Enright, G.D.; Villeneuve, D.M.; Dunn., J. and Baldis, H.A.; J. Opt. Soc. Am.; B8, 2047(1991).
3. Elton, R.C.; Lec, T.N. and Molander, W.A.; J. Opt. Soc. Am.; B4, 539(1987).
4. Mclean, E.A.; Lee, T.N.; Stamper, J.A.; Manka, C.K. and Griem, H.R.; J. Opt. Soc. Am.; B9, 350(1992).
5. Burnett, N.H. and Corkum, P.B.; J. Opt. Soc. Am.; B6, 1195(1989).

ISSN:0975-3583,0976-2833 VOL12,ISSUE02,2021

6. Penetrante, B.P. and Bardsley, J.N.; Phys. Rev. A43, 3100 (1991).
7. Lemoff, C.P.J. Barty and Harn,s, S.E.; Opt. Letters., 19, 569 (1994).
8. Nagata, Y.; Toyada, K; Phys. Rev. Letters., 71, 3774 (1993).
9. Eder, D.C.; Amendt, P.; Dasilva, L.B.; London, R.A.; Mac Gowan, B.J. Mathews, D.L.; Penetvante, B.M. Rosen, M.D.; Wilks, S.C.; Donnelly, T.D.; Falcone, R.W. and Strobel, G.L.; Phys. Plasmas, 1, 1744 (1994).
10. Ozaki, T. Kuroda, H.; Phys. Rev. E 51. R 24 (1995).
11. Siegman, A.E.; Lasers, University Science Book, Mill Valley CA (1998).
12. Saha, M.N. & Srivastova, B.N.; Atreatise on heat Twelve ed., pp. 655-663 (1988).
13. Venkataraman, G.; Saha and his formula, University Press. Pp. 41-61 (1995).
14. Boruah, A.G.; Propagation of a laser beam through disturbed environment. Ph.D. Thesis, pp. 166-130 (1995).
15. Chichkov, B.N.; Egbent, A.; Eichmann, H.; Momma, C.; Nolte, S.; Phys. Rev.; A, 1629 (1995).
16. Borah Ritumoni and Baruah G.D., Paramanu(Ind.Acad. Sc.) 55,1(2000) Asian Chem. Letts. 3, 63(1999)
17. Khnikar L. and Baruah G.D., Asian Chem. Lett. 3, 96 (1999).
18. Rocca J.J.. Moreno C.H., Marconi M.C. and Kanizay K., Opt. Lett. 24,420 (1999); Rocca JJ. et al. in X-Ray Lasers edited by Kale Y., Takuma II. and Daido 11 (Institute of Physics, University of Berkshire, Reading, 1 999).
19. Tomasini R., Lowenthal F. and Balmer .I.E., in X-ray Lasers 1998, Proceedings of the 6th International Conference on X-ray Lasers, edited by KatoY, Takuma H.and Daido H., (Institute of Physics, University of Berkshire, Reading,1 999);Tommasini R., Lowenthal F. and Balmer .I.E., Phvs. Rev.), 59, 1577(1999).
20. Matthews D., Hagelstein P., Rosen M., Kauffman R.L., Lee R., Wang C, Medecki IE, Cambell E.M., Ccglio N., Leipelt G., Lee P., Drake P., Pleasance L, Seppla L., Hatcher W., Ranibach G., Hawryluk A., Heaton G., Price R, Ozarski R., Speck R., Manes K., Underwood J., Toor A., Weaver T., Coleman L., DeMartini B., Auerbach .1., Turner R., Zagotta W., Hailey C, Eckart M., Hazi A., Whittrn B., Charatis G., Rockett P., Busch G., Sullivan D., Shephard C, Johnson R., Macgowan B., Burkhatter P., in Laser Techniques in the Extreme Ultraviolet, eds. Harris S.E and Lucartorto T.B, Amer. Just. Phys. Conf. Proc.. No.l 19, p.25(1984); Matthews D.L., Cambell E.M., Estabrook K., Hatch W., Kauffman R.L., Lee R.W. and Wang C.L., Appl. Phvs. I.ett.45, 226 (1984).
21. Silfvast W.T., in “Laser Fundamentals” (Cambridge University Press, Cambridge, 1996) p.256.
22. Elton R.C., X-Ray Lasers (Academic, Boston, M.A, 1990).
23. Khanikar L. and Baruah G.D., Asian J. Phys. 9(2), 455(2000)(Proceeclings of the International Conference on Lasers and Their Applications, edited by Rastogi V.K. and Baruah G.D.(Department of Physics, St. Joseph's College(Autonomous), Tiruchirapallii, India, March 01-04, 2000).

ISSN:0975-3583,0976-2833 VOL12,ISSUE02,2021

24. X-Ray Lasers, edited by Eder D.C. and Matthews D.L. (AIP, New York, 1994).