Assessing Technical Efficiency in Policing: An Application of Data Envelopment Analysis (DEA)

Dr. Bandi Ramanjineyulu

Manager, Clinical Statistics, GlaxoSmithKline, Bangalore, Karnataka, India

Email id : ramanji.bandi@gmail.com

ABSTRACT

This research explores the measurement of police technical efficiency using a Data Envelopment Analysis (DEA) approach. The study aims to provide a comprehensive assessment of the operational performance of police forces, considering multiple inputs and outputs. By employing DEA, a non-parametric method, we evaluate the relative efficiency of different police departments, identifying best practices and areas for improvement. The research contributes to the field by offering a quantitative framework for assessing the technical efficiency of law enforcement agencies, thereby facilitating evidence-based decision-making and resource allocation. Through an analysis of various police departments, this study seeks to enhance our understanding of the factors influencing police efficiency and inform strategies for optimizing law enforcement operation

I. INTRODUCTION:

Data envelopment deals with measuring input and output productive efficiencies of decision-making units (DMUs) which compete with each other in a criminal justice system.

Crime is an integral part of society. A number of socio-economic factors are believed to induce individuals to commit crime in crime analysis, we come across good and bad outputs if a crime is committed and if it is reported then investigation follows.

The decision-making units (DMUs) are ranked by using the DEA. For calculation of efficiency of DMU, shepard’s output distance function – free

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disposability outputs are used. The evolution of output technical efficiencies leads to rank the DMUs. If the occurring in ranking of technically efficient DMUs, we resolve the tie by means of own and cross efficiencies of those DMUs.

II. METHODOLOGY:

POLICE OUTPUT EFFICIENCY MEASUREMENT:

The output sets, equivalently the production possibility sets are used to measure output oriented productive efficiency. In the absence of undesirable outputs, to quantity the extent of output efficiency all outputs are attempted to reduce radially, holding the input vector constant. If no reduction is possible the state of the police is output efficient, otherwise inefficient.

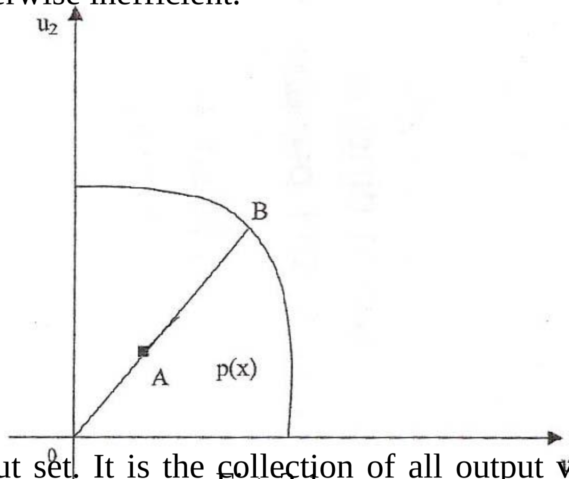

Fig. 2.1

- P (x) is output set. It is the collection of all output vectors producible by the input vector x
- $u \in R_2^+$
- $A(u_1^A, u_2^A)$. The producer who operates at A is inefficient since, $\left(u_1^A, u_2^A\right) \in p\left(x\right)$ is not a boundary point.
- To gain output technical efficiency further augmentation of outputs is desired from A to B, where, $B : B(u_1^B, u_2^B)$
- The departure of A from the boundary point is measured by the output distance function.

  $$D\left(x,u\right) = \frac{OA}{OB}$$

  $$O \le D\left(x,u\right) \le 1$$

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- An output distance function can be related with output level set as follows:
  $P(X) = \{u : D(x,u) \le 1\}$

In radial output technical efficiency measurement, the efficiency measures that we can obtain are,

- Pure output technical efficiency
- Overall output technical efficiency
- Revenue efficiency
- Allocative efficiency

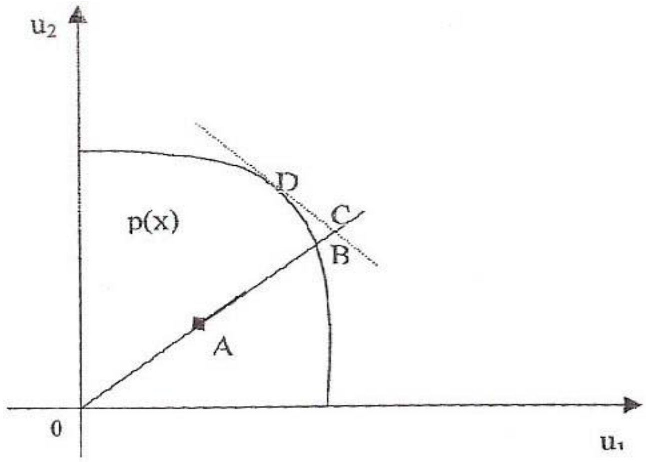

Fig. 2.2

- $u \in R_2^+$
- $P(x)$ is the output level set that admits constant returns to scale
- The state of police that operates at A is technically inefficient.
- To attain output technical efficiency, further output augmentation is required.
- Output overall technical efficiency: $\frac{OA}{OB}$
- Revenue at D is equal to revenue at C
- Output revenue efficiency: $\frac{OA}{OC}$

The output revenue efficiency can be multiplicatively decomposed into overall output technical and allocative efficiencies.

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$$\frac{OA}{OC} = \frac{OA}{OB} X \frac{OB}{OC}$$

Output allocative efficiency is, $\frac{OB}{OC}$

If all police outputs are good, returns to scale are constant, police outputs are freely disposable.

III. DIRECTIONAL DISTANCE FUNCTION (DDF):

Technical Efficiency can be measured by Directional Distance function which is additive in nature compared to Farrell's Technical Efficiency that is multiplicative. The DDF is defined as

$$D\left(x, u; g_x, g_y\right) = \underset{\delta}{Sup} \left\{\delta : \left(x - \delta g_x, x + \delta g_y\right) \in GR\right\} \quad (3.1)$$

- g_x and g_y are directional vectors for x and y
- $g_x \in R_n^+$
- $g_x \in R_m^+$
- GR is the production possibility set.

The Directional Distance function simultaneously enquires reduction of inputs and augmentation of outputs.

The following are some of the structural properties of the Directional Distance functions.

(i) $$D\left(x - \alpha g_x, y + \alpha g_y; g_x, g_y\right) = D\left(x, y;\ g_y, g_y\right) - \alpha, \alpha \in R \quad (3.2)$$

$$D\left(x - \alpha g_x, y + \alpha g_y; g_x, g_y\right) = \underset{\delta}{Sup} \left\{\delta : \left(x - \delta g_x - \alpha g_x, y + \delta g_y + \alpha g_y\right)\right\} \in GR$$

$$= \underset{\delta}{Sup} \left\{\delta : \left(x - \left(\alpha + \delta\right) g_x, y + \left(\alpha - \delta\right) g_y\right)\right\} \in GR$$

$$= \underset{\delta}{Sup} \left\{\alpha + \delta : \left(x - \left(\alpha + \delta\right) g_x, u + \left(\alpha + \delta\right) g_u\right) \in GR\right\} - \alpha$$

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$$= D\left(x, u; g_x, g_y\right) - \alpha$$

(ii) $$D\left(x, u; \lambda g_x, \lambda g_y\right) = \lambda^{-1} D\left(x, u; g_x, g_y\right) \quad (3.3)$$

$$D\left(x, u; \lambda g_x, \lambda g_y\right) = \underset{\delta}{Sup}\left\{\delta : \left(x - \delta\lambda g_x, u + \delta\lambda g_y\right) \in GR\right\}$$

$$= \lambda^{-1} \underset{\lambda\delta}{Sup}\left\{\delta\lambda : \left(x - \delta\lambda g_x, u + \delta\lambda g_y\right) \in GR\right\}$$

$$= \lambda^{-1} \underset{\bar{\delta}}{Sup}\left\{\bar{\delta} : \left(x - \bar{\delta} g_x, y + \bar{\delta} g_y\right) \in GR\right\}$$

$$= \lambda^{-1} D\left(x, u, \lambda g_x, \lambda g_y\right)$$

$$x^1 \geq x \Rightarrow D\left(x^1, u, g_x, g_y\right) \leq D\left(x, u, g_x, g_y\right) \quad (3.4)$$

(iii) $$x^1 \geq x \Rightarrow P(x) \subseteq P\left(x^1\right)$$

$$\Rightarrow G(x, u) \subseteq G(x^1, u)$$

$$\underset{\delta}{Sup}\left\{\delta : \left(x - \delta g_x, y + \delta g_y\right) \in GR\right\} \leq \underset{\delta}{Sup}\left\{\delta : \left(x^1 - \delta g_x, y + \delta g_y\right) \in GR\right\}$$

$$D\left(x, y; g_x, g_y\right) \leq D\left(x^1, y; g_x, g_y\right)$$

(iv) $$y^1 \geq y \Rightarrow D\left(x, y^1; g_x, g_y\right) \leq D\left(x, y; g_x, g_y\right) \quad (3.\ 5)$$

$$y \leq y^1 \in P(x) \Rightarrow y \in P(x) \quad GR\,(x, y) \subseteq GR\,(x, y^1)$$

$$\underset{\delta}{Sup}\left\{\delta : \left(x - g_x\delta, y + g_y\delta\right) \in GR\right\} \leq \underset{\delta}{Sup}\left\{\delta : \left(x - \delta g_x, y + \delta g_y\right) \in GR\right\}$$

$$D\left(x, y : g_x, g_y\right) \leq D\left(x, y^1 : g_x, g_y\right)$$

If returns to scale are constant

(v) $$D\left(\lambda x, \lambda y; g_x, g_y\right) = \lambda D\left(x, y : g_x, g_y\right) \quad (3.6)$$

$$D\left(\lambda x, \lambda y; g_x, g_y\right) = \underset{\delta}{Sup}\left\{\delta : \left(\lambda x - \delta g_x, \lambda y + \delta g_y\right) \in GR\right\}$$

$$= \underset{\delta}{Sup}\left\{\delta : \lambda\left[x - \frac{\delta}{\lambda} g_x, y + \frac{\delta}{\lambda} g_y\right] \in GR\right\}$$

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$$= \underset{\delta}{Sup}\left\{\delta : \left[x - \frac{\delta}{\lambda} g_x, y + \frac{\delta}{\lambda} g_y\right] \in \frac{1}{\lambda} GR\right\}$$

$$= \underset{\delta}{Sup}\left\{\delta : \left[x - \frac{\delta}{\lambda} g_x, y + \frac{\delta}{\lambda} g_y\right] \in GR\right\}$$

$$= \lambda \underset{\frac{\delta}{\lambda}}{Sup}\left\{\frac{\delta}{\lambda} : \left[x - \frac{\delta}{\lambda} g_x, y + \frac{\delta}{\lambda} g_y\right]\right\}$$

$$= \lambda D\left(x, y ; g_x, g_y\right)$$

REVENUE EFFICIENCY INDICATOR:

Potential Revenue	:	R (x, r)
Observed Revenue	:	ry
Directional Vector	:	g_y

Revenue efficiency indicator is defined as $R.I\left(x, y, r ; g_y\right) = \frac{R(x, r) - ry}{rg_y}$ (3.7)

For police of any state, price vector r is unavailable for police outputs as measured by the proportion of crimes for which charge sheets are filled to the total number of reported crimes in each category of crimes.

However, for property crimes a proxy for police output price can be derived. Value of property recovered may be used as police output price for property crimes.

IV. RELATIONSHIP BETWEEN SHEPHARD'S OUTPUT DISTANCE FUNCTION AND DIRECTIONAL OUTPUT DISTANCE FUNCTION:

Consider, $D_0\left(x, y / y\right) = \underset{\delta}{Sup}\left\{\delta : \left(y + \delta y\right) \in P\left(x\right)\right\}$

$$= \underset{\delta}{Sup}\left\{\delta : \left(1 + \delta\right) y \in P\left(x\right)\right\}$$

$$= \underset{(1+\delta)}{Sup}\left\{\left(1 + \delta\right) : \left(1 + \delta\right) y \in P\left(x\right)\right\} - 1$$

$$= \underset{\delta'}{Sup}\left\{\delta' : \delta' y \in P\left(x\right)\right\} - 1$$

$$D_0\left(x, y / y\right) = \left[D_0\left(x, u\right)\right] - 1 \quad (4.1)$$

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Thus, if g_y = y, the directional and Shephards output distance functions are related as follows:

$$D_0(x, y/y) = \frac{1}{\left[D_0(x,u)^{-1}\right]} - 1 \quad (4.2)$$

GRAPHIC REPRESENTATION OF DIRECTIONAL OUTPUT DISTANCE FUNCTION:

- $x \in R^+$
- $y \in R_2^+$
- $y = (y_1, y_2), y_1$ *and* y_2 are good outputs.

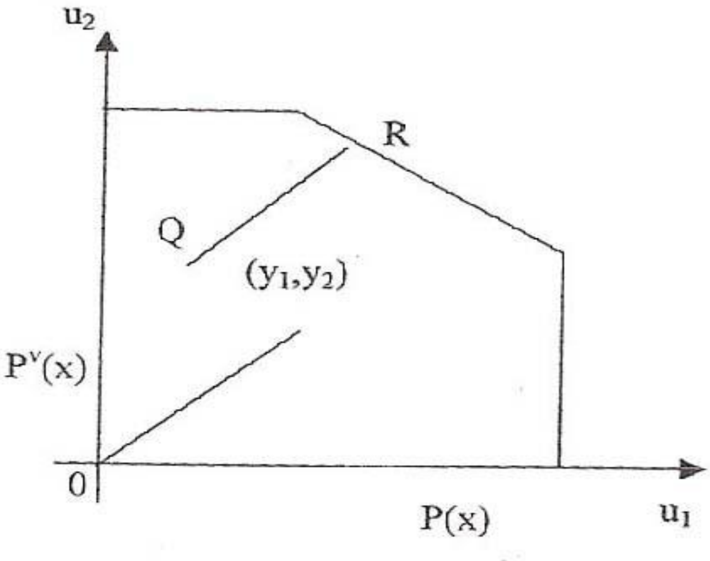

Fig. 4.1

- P(x) is output level set.
- $y = (y_1, y_2)$ is output vector.
- First and Second outputs are measured along horizontal and vertical axis respectively.
- y is inefficient output vector.
- The ray that emanates from origin is the directional vector, $g_y = (g_{y_1}, g_{y_2})$

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- The inefficient output vector is projected to the efficient subset of P(x) in the direction of directional vector g_y.
- $Q\left(y_1, y_2\right)$
- $R\left(y_1 + D\left(x, y/g_y\right),\ y_2 + D_2\left(x, y/g_y\right)\right)$

The directional output distance functions can be used to measure simultaneous additional augmentation of good outputs and reduction of bad outputs; to estimate the appropriate directions output distance function.

Let y and u denote the vectors of good and bad outputs respectively. The directional distance function that expands good outputs (y) and contracts bad outputs (u) may be expressed as follows:

$$D_0\left(x, y, u / g_y, g_u\right) = \underset{\delta}{Max}\left\{\delta : \left(y + \delta g_y, u - \delta g_x\right) \in p(x)\right\} \qquad (4.3)$$

V. EMPIRICAL INVESTIGATION:

The police personnel of one Indian state serves as one decision making unit. As such we have 28DMUs. There are

1. Andhra Pradesh(AP)
2. Arunachal Pradesh
3. Assam
4. Bihar
5. Chhattisgarh

6. Goa
7. Gujarat
8. Haryana
9. Himachal Pradesh
10. Jammu and Kashmir

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11. Jharkhand
12. Karnataka
13. Kerala
14. Madhya Pradesh
15. Maharashtra
16. Manipur
17. Meghalaya
18. Mizoram
19. Nagaland
20. Orissa
21. Punjab
22. Rajasthan
23. Sikkim
24. Tamilnadu
25. Tripura
26. Uttar Pradesh
27. Uttaranchal
28. West Bengal

Thus, there are 28 decision making units which combine 3 inputs produce 5 outputs of which one output is bad.

Table-5.1

Police of	Shephard's Output Distance Function	Output Technical Efficiency
Andhra Pradesh	0.394	2.5381
Arunachal Pradesh	1.000	1.0000
Assam	1.000	1.0000
Bihar	0.335	2.9851

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Chhattisgarh	1.000	1.0000
Goa	1.000	1.0000
Gujarat	0.404	2.4752
Haryana	1.000	1.0000
Himachal Pradesh	1.000	1.0000
Jammu and Kashmir	0.408	2.3923
Jharkhand	1.000	1.0000
Karnataka	0.338	2.9589
Kerala	0.396	2.5252
Madhya Pradesh	0.223	4.2918
Maharashtra	0.227	3.6101
Manipur	0.234	4.2735
Meghalaya	1.000	1.0000
Mizoram	1.000	1.0000
Nagaland	1.000	1.0000
Orissa	0.849	1.1779
Punjab	0.352	2.8409
Rajasthan	0.632	1.5823
Sikkim	1.000	1.0000
Tamil Nadu	0.314	3.1847
Tripura	1.000	1.0000
Uttar Pradesh	0.206	4.8544
Uttaranchal	1.000	1.0000
West Bengal	0.296	3.3784

There are 13 States of police that are output technical efficient. Decision making units for which shephard’s output distance function takes unit value are output technical efficient.

15 Output technical inefficiency states are ranked according to the output technical efficiency whereas for 13 output technical efficient states has unique output technical efficient states has unique output technical efficiency to dissolve the tie among the output technical efficiency by peer count.

- For an efficient unit peer count itself.

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- Peers and ideal production units for an inefficient DMU.
- More a police states appears in peer lists of inefficient unit better is its status among technically efficient states of police.

Table-5. 2

DMU	Peer Count
1	0
2	9
3	1
4	0
5	7
6	10
7	0
8	3
9	4
10	0
11	3
12	0
13	0
14	0
15	0
16	0
17	3
18	4
19	1
20	0
21	0
22	0
23	7
24	0
25	2
26	0
27	14
28	0

For Fifteen DMUs the Peer Count is zero. We have already witnessed that these are inefficient DMUs.

For Output technically efficient DMUs the Peer Count is a minimum of one. DMU-27 appears 14 times in the peer list of 15 inefficient decision-making units. As

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such in efficient ratings it occupies first position. DMU-27 represents the police of the state Uttaranchal.

Peer Count Summary is essential to rank the DMUs that are technically efficient. However, if we fail to resolve the rank problem by exercising peer counts (when tie occurs), we may resort to cross efficiency.

In the above table

1. DMU-3 and DMU-19 has the same peer count '1'.
2. DMU-8, DMU-11 and DMU-17 has the same peer count as '3'.
3. DMU-5 and DMU-23 has the peer count '7' and
4. DMU-9 and DMU-18 has peer count '4'. These ties are resolved by using the cross efficiency.

The Efficiency of 28 Police States of India are ranked using Own & Mean of Cross Efficiencies as follows:

Table-5.3

DMU No.	DMU Name	Own Efficiency	Mean of Cross Efficiency	Peer Count	Rank
27	Uttaranchal	1		14	1
6	Goa	1		10	2
2	Arunachal Pradesh	1		9	3
23	Sikkim	1	1.2999	7	4
5	Chhattisgarh	1	3.5661	7	5
18	Mizoram	1	1.1241	4	6
9	Himachal Pradesh	1	1.3831	4	7

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17	Meghalaya	1	1.5131	3	8
8	Haryana	1	2.2677	3	9
11	Jharkhand	1	3.3018	3	10
25	Tripura	1		2	11
19	Nagaland	1	1.4718	1	12
3	Assam	1	6.6244	1	13
20	Orissa	1.1779			14
22	Rajasthan	1.5823			15
23	Jammu and Kashmir	2.3923			16
7	Gujarat	2.4752			17
13	Kerala	2.5252			18
1	Andhra Pradesh	2.5381			19
21	Punjab	2.8409			20
12	Karnataka	2.9589			21
4	Bihar	2.9851			22
24	Tamil Nadu	3.1847			23
28	West Bengal	3.3784			24
15	Maharashtra	3.6101			25
16	Manipur	4.2735			26
17	Madhya Pradesh	4.2918			27
26	Uttar Pradesh	4.8544			28

VI. CONCLUSIONS:

For 28 Police States assuming that the return to scale are constant linear programming problem solved.

The police organizations of 13 states are found technically efficient. Among the rest of 15 police organizations it is observed that there is a significant variation in output technical efficiency. Efficient states are those that are relatively smaller in area and density of population. In some of these states bad output viz., custodial crimes are not registered.

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To rank the 13 efficient decision making units performed peer analysis. If an efficient DMU appears in the peer list of inefficient DMUs than another DMU then the foemen DMU is considered to be more efficient than the later if peer analysis, fails to resolve a tie among efficient units, one may resort to cross efficiency analysis that requires to solve appropriate dual linear programming problems one time for one DMU.

The peer analysis failed to resolve tie between DMU5 and DMU23; DMU9 and DMU18; DMU8, DMU11 and DMU17; DMU3 and DMU19 through cross efficiency evaluation the ties are resolved for tied efficient DMUs these police organizations are ranked as shown below.

Table.1

States	Rank
Uttaranchal	1
Goa	2
Arunachal Pradesh	3
Sikkim	4
Chhattisgarh	5
Mizoram	6
Himachal Pradesh	7
Meghalaya	8
Haryana	9
Jharkhand	10
Tripura	11
Nagaland	12
Assam	13

The rest of police organizations of 15 states that are inefficient are ranked according to the good output losses they suffer from. The fifteen DMUs are ranked follows:

Table.2

States	Rank
Orissa	14
Rajasthan	15

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Jammu and Kashmir	16
Gujarat	17
Kerala	18
Andhra Pradesh	19
Punjab	20
Karnataka	21
Bihar	22
Tamilnadu	23
West Bengal	24
Maharashtra	25
Manipur	26
Madhya Pradesh	27
Uttar Pradesh	28

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