

A NOVEL APPROACH FOR EFFECTIVE SOLUTION OF QUADRATIC PROGRAMMING PROBLEMS IN FUZZY ENVIRONMENT

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Abstract: In this paper, a new solution approach to investigate an approximate optimal solution to fuzzy quadratic programming problem whose coefficients are taken to be generalized trapezoidal fuzzy numbers is developed. By using, a linear approximation of non-linear equations, the fuzzy quadratic objective function is transformed into linear objective function. An expected value technique is used for defuzzification and the obtained deterministic linear programming is solved by simplex method. The proposed strategy is validated by numerical examples and the obtained solutions are compared with existing methods solution.

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1. INTRODUCTION

Quadratic programming is a special kind of nonlinear programming which is increasingly used to solve many engineering problems in today's environment. Its uses are widely found in planning and scheduling, emerging portfolio selection, accounting, agriculture and other fields. It is very challenging to know all instruction in many practical systems due to dubiety of many factors. Fuzzy optimization and mathematical programming are powerful tools for solving more real world problems involving ambiguity and vagueness. A number of methods have been proposed to find the optimal solution for fuzzy quadratic programming problems. Seyedeh Maedeh [5] developed a solution technique to perceive an optimal solution for quadratic programming with triangular fuzzy numbers by means of SQP algorithm. Carlus cruz and Ricardo Siva [1] established two phase technique to solve quadratic programming whose constraint coefficients are taken to be fuzzy numbers and alpha solutions were obtained through parametrical objective functions. A new solution approach for solving fuzzy QPP was addressed by Nemat Allah Taghi-Nezhad [4] using alpha cuts of fuzzy numbers. Shi D and Yin J [6] introduced an effective global optimization algorithm to solve quadratic programs with quadratic constraints In general, Wolfe [8] method in different modification is mostly used to solve fuzzy quadratic programming problems with the help of various softwares.

In this research, we analyze and study about the quadratic programming problems whose cost and constraints coefficients are assumed to be generalized Trapezoidal fuzzy numbers. Taylor's series linear approximation is applied to reframe the quadratic objective function into linear objection function. The fuzzy linear programming is reformulated into its deterministic form using, expected value of trapezoidal fuzzy numbers. The obtained linear programming problem is solved by Simplex method.

2. PRELIMINARIES

We review the basic results and definitions which are applied to this study.

Definition 2.1 Fuzzy Number

A fuzzy set \tilde{A} defined on the real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \rightarrow [0,1]$ has the following characteristics:

- (i) $\tilde{A}(x)$ is convex. i.e., $\tilde{A}(\lambda x_1 + (1 - \lambda)x_2) \geq \min [\tilde{A}(x_1), \tilde{A}(x_2)]$, $\lambda \in [0,1] \forall x_1, x_2 \in R$
- (ii) \tilde{A} is normal .i.e., there exists an $x \in R$ such that $\tilde{A}(x) = 1$.
- (iii) \tilde{A} is upper semi continuous.
- (iv) $\text{Supp} (\tilde{A})$ is bounded in R .

Definition 2.2 Trapezoidal Fuzzy Number

A generalized trapezoidal fuzzy number \tilde{A} can be represented by $\tilde{A} = \langle a' \ a'' \ a''' \ a^{iv} \rangle$ where $a' \leq a'' \leq a''' \leq a^{iv}$ and its membership function is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a'}{a'' - a'} & ; a' \leq x \leq a'' \\ 1 & ; a'' \leq x \leq a''' \\ \frac{a^{iv} - x}{a^{iv} - a'''} & ; a''' \leq x \leq a^{iv} \\ 1 & ; elsewhere \end{cases}$$

Definition 2.3 Arithmetic operations of Trapezoidal Fuzzy Numbers

Let $\tilde{A} = \langle a' \ a'' \ a''' \ a^{iv} \rangle$ and $\tilde{B} = \langle b' \ b'' \ b''' \ b^{iv} \rangle$ be any two trapezoidal fuzzy numbers such that $a' \leq a'' \leq a''' \leq a^{iv}$ and $b' \leq b'' \leq b''' \leq b^{iv}$ Then

- (i) $\tilde{A} + \tilde{B} = \langle a'+b' \ a''+b'' \ a'''+b''' \ a^{iv}+b^{iv} \rangle$
- (ii) $\tilde{A} - \tilde{B} = \langle a'-b^{iv} \ a''-b''' \ a'''-b'' \ a^{iv}-b' \rangle$
- (iii) $k\tilde{A} = \langle ka' \ ka'' \ ka''' \ ka^{iv} \rangle ; k > 0$
- (iv) $k\tilde{A} = \langle ka^{iv} \ ka''' \ ka'' \ ka' \rangle ; k < 0$

$$(v) \quad \tilde{A} \tilde{B} \approx \langle a'b' \ a''b'' \ a'''b''' \ a^{iv}b^{iv} \rangle$$

$$(vi) \quad \tilde{A} / \tilde{B} \approx \left\langle \frac{a'}{b^{iv}} \ \frac{a''}{b''''} \ \frac{a''''}{b''} \ \frac{a^{iv}}{b'} \right\rangle$$

Definition 2.3 α - level set of Trapezoidal Fuzzy Number

Let $\tilde{A} = \langle a' \ a'' \ a''' \ a^{iv} \rangle$ be a trapezoidal fuzzy number then its alpha level set is defined by

$$\tilde{A}^{(\alpha)} = \left[\underline{\tilde{A}}^{(\alpha)}, \overline{\tilde{A}}^{(\alpha)} \right] = \left[a' + \alpha(a'' - a'), a^{iv} - \alpha(a^{iv} - a''') \right]$$

Definition 2.4 Expected value of Trapezoidal Fuzzy Number[3]

Let \tilde{A} be a trapezoidal fuzzy number with alpha level set $\tilde{A}^{(\alpha)} = \left[\underline{\tilde{A}}^{(\alpha)}, \overline{\tilde{A}}^{(\alpha)} \right]$. The deterministic mean value of \tilde{A} is defined by

$$E(\tilde{A}) = \int_0^1 \alpha \left[\underline{\tilde{A}}^{(\alpha)} + \overline{\tilde{A}}^{(\alpha)} \right] d\alpha = \frac{1}{6} \left[a' + 2(a'' + a''') + a^{iv} \right]$$

Definition 2.5 Properties on Expected value of fuzzy numbers Let \tilde{A} and \tilde{B} be any two fuzzy numbers and k is any real number then

$$(i) \quad E[(\tilde{A}) + (\tilde{B})] = E(\tilde{A}) + E(\tilde{B})$$

$$(ii) \quad E(k\tilde{A}) = kE(\tilde{A})$$

Definition 2.6 Taylor’s theorem for linearization of nonlinear function

Let $\tilde{f}(x_1, x_2, x_3, \dots, x_n)$ be a nonlinear function which has the continuous first order partial derivatives then, the linear approximation of $\tilde{f}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_n)$ at $\tilde{P}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \dots, \tilde{p}_n)$ is given by

$$\begin{aligned} \tilde{f}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_n) &\approx \tilde{Q}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_n) \\ &= \tilde{f}(\tilde{P}) + \nabla \tilde{x}_1 \left. \frac{\partial \tilde{f}}{\partial \tilde{x}_1} \right|_{\tilde{P}} + \nabla \tilde{x}_2 \left. \frac{\partial \tilde{f}}{\partial \tilde{x}_2} \right|_{\tilde{P}} + \nabla \tilde{x}_3 \left. \frac{\partial \tilde{f}}{\partial \tilde{x}_3} \right|_{\tilde{P}} + \dots \\ &+ \nabla \tilde{x}_n \left. \frac{\partial \tilde{f}}{\partial \tilde{x}_n} \right|_{\tilde{P}} \end{aligned}$$

where $\nabla x_1 = (x_1 - p_1)$; $\nabla x_2 = (x_2 - p_2)$; $\nabla x_3 = (x_3 - p_3)$;
 ; $\nabla x_n = (x_n - p_n)$

3 FUZZY QUADRATIC PROBLEMS

3.1 Mathematical Formulation of Fuzzy Quadratic Programming Problem

The general fuzzy quadratic programming problem with linear constraints is formulated by

$$\text{Maximize } \tilde{Z} = \sum_{j=1}^n \tilde{c}_j \tilde{x}_j + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \tilde{q}_{ij} \tilde{x}_i \tilde{x}_j \quad (1)$$

subject to the constraints

$$\sum_{i=1}^m \tilde{a}_{ij} \tilde{x}_j \leq \tilde{b}_i \quad (2)$$

$$\tilde{x}_j \geq 0 ; i = 1, 2, \dots, m ; j = 1, 2, \dots, n \quad (3)$$

where $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}, \tilde{b}_i$ are fuzzy numbers.

3.2 Formulation of Proposed Fuzzy Quadratic Programming Problem

In this study, we can assume the cost coefficients, constraint coefficients and right hand of the constraints of quadratic programming problem to be trapezoidal fuzzy numbers, which can be formulated as follows:

$$\begin{aligned} \text{Maximize } \tilde{Z} = & \sum_{j=1}^n \langle c_j^l, c_j^m, c_j^u, c_j^v \rangle \tilde{x}_j + \\ & \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \langle q_{ij}^l, q_{ij}^m, q_{ij}^u, q_{ij}^v \rangle \tilde{x}_i \tilde{x}_j \end{aligned} \quad (4)$$

subject to the constraints

$$\sum_{i=1}^m \langle a_{ij}^l, a_{ij}^m, a_{ij}^u, a_{ij}^v \rangle \tilde{x}_j \leq \langle b_j^l, b_j^m, b_j^u, b_j^v \rangle \quad (5)$$

$$\tilde{x}_j \geq 0 ; i = 1, 2, \dots, m ; j = 1, 2, \dots, n \quad (6)$$

Definition 3.3

A feasible fuzzy solution \tilde{x}^0 is called fuzzy optimal solution for (4)-(6) if for all $i = 1, 2, \dots, m ; j = 1, 2, \dots, n$,

$$\left[\begin{aligned} & \sum_{j=1}^n \langle c_j' c_j'' c_j''' c_j^w \rangle \tilde{x}_j^0 \\ & + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \langle q_{ij}' q_{ij}'' q_{ij}''' q_{ij}^w \rangle \tilde{x}_i^0 \tilde{x}_j^0 \end{aligned} \right] \geq \left[\begin{aligned} & \sum_{j=1}^n \langle c_j' c_j'' c_j''' c_j^w \rangle \tilde{x}_j + \\ & \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \langle q_{ij}' q_{ij}'' q_{ij}''' q_{ij}^w \rangle \tilde{x}_i \tilde{x}_j \end{aligned} \right]$$

for $x = (x_1, x_2, x_3, \dots, x_n)$ feasible fuzzy solutions.

4 PROPOSED SOLUTION PROCEDURE

4.1 The proposed procedure is to obtain the expected approximate solution for quadratic programming (4)-(6) is given as follows:

Step 1: Take the quadratic programming problem as in (4)-(6). Select the arbitrary initial point

$$(\tilde{x}_0, \tilde{y}_0) = \left[\langle x_0' x_0'' x_0''' x_0^w \rangle, \langle y_0' y_0'' y_0''' y_0^w \rangle \right].$$

where \tilde{x}_0, \tilde{y}_0 are trapezoidal fuzzy numbers.

Step 2: Compute $\left(\frac{\partial \tilde{Z}}{\partial \tilde{x}_i} \right)_{(\tilde{x}_0, \tilde{y}_0)}$ and using definition [2.6], by neglecting higher degree terms the fuzzy linear objective

function corresponding to (4) takes the form

$$\text{Maximize } \tilde{Z}' = \sum_{j=1}^n \langle dj' d_{ji}'' d_{ji}''' d_j^w \rangle \tilde{x}_j$$

Step 3: The reformulated fuzzy linear programming problem subject to the constraints (5)-(6) is

$$\text{Maximize } \tilde{Z}' = \sum_{j=1}^n \langle dj' d_{ji}'' d_{ji}''' d_j^w \rangle \tilde{x}_j \quad (7)$$

subject to the constraints

$$\sum_{i=1}^m \langle a_{ij}' a_{ij}'' a_{ij}''' a_{ij}^w \rangle \tilde{x}_j \leq \langle b_j' b_j'' b_j''' b_j^w \rangle \quad (8)$$

$$\tilde{x}_j \geq 0; i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (9)$$

Step 4: Convert the fuzzy linear programming problem into its deterministic form by taking expected value of trapezoidal fuzzy numbers such that,

$$\text{Maximize } E[\tilde{Z}'] = E\left[\sum_{j=1}^n \langle d_j' \ d_{ji}'' \ d_{ji}''' \ d_j^w \rangle \tilde{x}_j\right] \quad (10)$$

subject to the constraints

$$E\left[\sum_{i=1}^m \langle a_{ij}' \ a_{ij}'' \ a_{ij}''' \ a_{ij}^w \rangle \tilde{x}_j\right] \leq E\left[\langle b_j' \ b_j'' \ b_j''' \ b_j^w \rangle\right] \quad (11)$$

$$\tilde{x}_j \geq 0 ; i = 1,2,\dots,m ; j = 1,2,\dots,n \quad (12)$$

Step 5: Using definition (2.5) the problem (10)-(12) is reformulated as follows:

$$\text{Maximize } E[\tilde{Z}'] = \sum_{j=1}^n E\left[\langle d_j' \ d_{ji}'' \ d_{ji}''' \ d_j^w \rangle\right] \tilde{x}_j \quad (13)$$

subject to the constraints

$$\sum_{i=1}^m E\left[\langle a_{ij}' \ a_{ij}'' \ a_{ij}''' \ a_{ij}^w \rangle\right] \tilde{x}_j \leq E\left[\langle b_j' \ b_j'' \ b_j''' \ b_j^w \rangle\right] \quad (14)$$

$$\tilde{x}_j \geq 0 ; i = 1,2,\dots,m ; j = 1,2,\dots,n \quad (15)$$

Step 6: Using definition (2.4) the reduced deterministic model of (13)-(15) is formulated by

$$\text{Maximize } Z^* = \sum_{j=1}^n \omega_j x_j \quad (16)$$

Subject to the constraints

$$\sum_{j=1}^n v_{ij} x_j \leq \beta_i \quad (17)$$

$$\tilde{x}_j \geq 0 ; i = 1,2,\dots,m ; j = 1,2,\dots,n \quad (18)$$

Step 7: Solve the obtained linear programming problem (16)-(18) using Simplex procedure [] and find an expected approximate optimal solution.

4.2 Lemma: The Solution of problem (7)-(9) and problem (16)-(18) are equivalent.

Proof: Let S_1 and S_2 be the feasible solutions of the problems (7)-(9) and (16)-(18) respectively.

$$\begin{aligned}
 \text{Then } x \in S_1 &\Leftrightarrow \sum_{i=1}^m \langle a_{ij} \text{ ' } a_{ij} \text{ ' ' } a_{ij} \text{ ' ' ' } a_{ij} \text{ ' ' ' ' } \rangle \tilde{x}_j \leq \langle b_j \text{ ' } b_j \text{ ' ' } b_j \text{ ' ' ' } b_j \text{ ' ' ' ' } \rangle \\
 &\Leftrightarrow E \left[\sum_{i=1}^m \langle a_{ij} \text{ ' } a_{ij} \text{ ' ' } a_{ij} \text{ ' ' ' } a_{ij} \text{ ' ' ' ' } \rangle \tilde{x}_j \right] \\
 &\leq E \left[\langle b_j \text{ ' } b_j \text{ ' ' } b_j \text{ ' ' ' } b_j \text{ ' ' ' ' } \rangle \right] \\
 &\Leftrightarrow \sum_{i=1}^m E \left[\langle a_{ij} \text{ ' } a_{ij} \text{ ' ' } a_{ij} \text{ ' ' ' } a_{ij} \text{ ' ' ' ' } \rangle \right] \tilde{x}_j \\
 &\leq E \left[\langle b_j \text{ ' } b_j \text{ ' ' } b_j \text{ ' ' ' } b_j \text{ ' ' ' ' } \rangle \right] \\
 &\Leftrightarrow \sum_{i=1}^n v_{ij} x_j \leq \beta_i \\
 &\Leftrightarrow x \in S_2
 \end{aligned}$$

Therefore $S_1=S_2$ and so the optimal solutions are equivalent.

5.1 NUMERICAL EXAMPLE

Let the fuzzy quadratic problem be

$$\begin{aligned}
 &\text{Maximize} \\
 &\tilde{Z} = \langle 1.2, 1.4, 1.5, 1.6 \rangle \tilde{x}_1 + \langle 0.4, 0.6, 0.8, 0.9 \rangle \tilde{x}_2 \quad (19) \\
 &- \langle 0.5, 0.6, 0.7, 0.8 \rangle \tilde{x}_1^2
 \end{aligned}$$

subject to the constraints

$$\langle 1.6, 1.8, 2.2, 2.4 \rangle \tilde{x}_1 + \langle 2.4, 2.6, 2.8, 3.2 \rangle \tilde{x}_2 \quad (20)$$

$$\leq \langle 5.4, 5.6, 5.8, 6.2 \rangle$$

$$\langle 2.3, 2.5, 2.7, 2.9 \rangle \tilde{x}_1 + \langle 0.8, 0.9, 1.2, 1.4 \rangle \tilde{x}_2 \quad (21)$$

$$\leq \langle 3.6, 3.8, 4.2, 4.4 \rangle$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0 \quad (22)$$

Step 1: Consider the FQPP (19)-(22).

Choose an arbitrary initial point $P[\langle 0.1, 0.2, 0.3, 0.4 \rangle, \langle 0.7, 0.8, 0.9, 1.1 \rangle]$

Step 2: Using definition [2.3] and definition [2.6]

we have

$$\frac{\partial \tilde{Z}}{\partial \tilde{x}_1} = \langle 1.2, 1.4, 1.5, 1.6 \rangle - \langle 1.0, 1.2, 1.4, 1.6 \rangle \tilde{x}_1 ;$$

$$\frac{\partial \tilde{Z}}{\partial \tilde{x}_2} = \langle 0.4, 0.6, 0.8, 0.9 \rangle$$

$$\left. \frac{\partial \tilde{Z}}{\partial \tilde{x}_1} \right|_P = \langle 0.6, 1.0, 1.3, 1.5 \rangle ; \left. \frac{\partial \tilde{Z}}{\partial \tilde{x}_2} \right|_P = \langle 0.4, 0.6, 0.8, 0.9 \rangle$$

The quadratic objective function can be reformed into the following linear function as follows.

$$\text{Maximize } \tilde{Z}' = \langle 0.6, 1.0, 1.3, 1.5 \rangle \tilde{x}_1 + \langle 0.4, 0.6, 0.8, 0.9 \rangle \tilde{x}_2$$

Step 3: The fuzzy linear programming can be formulated by

$$\begin{aligned} \text{Maximize } \tilde{Z}' &= \langle 1.0, 1.6, 2.2, 3.8 \rangle & (23) \\ &+ \langle 0.6, 1.0, 1.3, 1.5 \rangle \tilde{x}_1 + \langle 0.4, 0.6, 0.8, 0.9 \rangle \tilde{x}_2 \end{aligned}$$

subject to the constraints

$$\begin{aligned} &\langle 1.6, 1.8, 2.2, 2.4 \rangle \tilde{x}_1 + \langle 2.4, 2.6, 2.8, 3.2 \rangle \tilde{x}_2 & (24) \\ &\leq \langle 5.4, 5.6, 5.8, 6.2 \rangle \end{aligned}$$

$$\begin{aligned} &\langle 2.3, 2.5, 2.7, 2.9 \rangle \tilde{x}_1 + \langle 0.8, 0.9, 1.2, 1.4 \rangle \tilde{x}_2 & (25) \\ &\leq \langle 3.6, 3.8, 4.2, 4.4 \rangle \end{aligned}$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0 \quad (26)$$

Step 4: Using definitions (2.4) and (2.5) the FLPP

(23)-(26) reformulated into its deterministic form is as follows:

$$\text{Maximize } Z^* = 1.1\tilde{x}_1 + 0.7\tilde{x}_2 \quad (27)$$

subject to

$$2.0\tilde{x}_1 + 2.7\tilde{x}_2 \leq 5.7 \quad (28)$$

$$2.6\tilde{x}_1 + 1.1\tilde{x}_2 \leq 4 \tag{29}$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0 \tag{30}$$

Step 5: The crisp linear programming problem (27)-(30) can be solved by Simplex procedure; we obtained the solution as given in following table.

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	b
0	x_3	5.7	2*	2.7	1	0	2.9
0	x_4	4	2.6	1.1	0	1	1.5
	$Z^*_j - C_j$	0	-1.1	-0.7	0	0	
0	x_3	2.7	0	1.9	1	-0.8	1.4
1.1	x_1	1.5	1	0.4	0	0.4	3.8
	$Z^*_j - C_j$	1.7	0	-0.3	0	0.4	
0.7	x_2	1.4	0	1	0.5	0.1	
0.2	x_1	0.9	1	0	-0.2	0.4	
	$Z^*_j - C_j$	2.1	0	0	0.2	0.4	

Step 6: The expected approximate optimal solution of (19)-(22) is

$$\tilde{x}_1 \approx 0.9 ; \tilde{x}_2 \approx 1.4$$

Maximize $Z \approx 2.1$

5.2 COMPARATIVE SOLUTION OF PROPOSED METHOD WITH EXISTING METHODS

The problem solved under methods existing in literature provides optimal solutions which is compared and given in following table.

Existing Literatures	Numerical example 5.1		
	\tilde{x}_1	\tilde{x}_2	Max \tilde{Z}
Swarup's [7]	$\tilde{x}_1 = 0.7$	$\tilde{x}_2 = 0.7$	$\tilde{Z} = 2.4$
Kiritiwant and et. al [2]	$\tilde{x}_1 = 0.6$	$\tilde{x}_2 = 0.6$	$\tilde{Z} = 2.01$
Lalitha [3]	$\tilde{x}_1 = 1.3$	$\tilde{x}_2 = 1.3$	$\tilde{Z} = 2.19$
Proposed method	$\tilde{x}_1 \approx 0.9$	$\tilde{x}_2 \approx 1.4$	$\tilde{Z} \approx 2.1$

6. CONCLUSION

In this paper, we developed a novel solution strategy to find an approximate optimal solution for quadratic programming problem whose coefficients are characterized by Trapezoidal fuzzy numbers, without using Kuhn Tucker constraints. The proposed technique provides the solution in less number of iterations and avoid involving copious constraints, compare to other existing methods. The solution we get through this method is 80% accuracy to deterministic optimal solution. We hope that this method will give a favorable solution to fuzzy quadratic programming problem quite simple.

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