

SUMMATION OF GENERALISED k - JACOBSTHAL NUMBERS

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ABSTRACT

In this paper we deliberate closed form of Generalised k -Jacobsthal Numbers.From that we deduce summation formula for -Jacobsthal, k -Jacobsthal Lucas, k -Jacobsthal Derived k -Jacobsthal ,Derived k - Jacobsthal Lucas numbers.

SUBJECT CLASSIFICATIONS: MSC[2010] :11B37,11B39,11B83.

1. INTRODUCTION

There are to many studies in the literature that concern about the special second order such as generalised k -Fibonacci, k -Lucas, k -Jacobsthal etc.In this paper we discuss about Summation of Generalized k -Jacobsthal Numbers. We have discuss it in three cases. . From that We retrieve the Sum formula of k -Jacobsthal, k -Jacobsthal Lucas, Derived k -Jacobsthal, and Derived k -Jacobsthal Lucas

NOTATIONS:- $J_{k,n}, \hat{J}_{k,n}, D\hat{J}_{k,n}, D\hat{c}_{k,n}$

2. GENERALISED k –JACOBSTHAL NUMBER:

Let k be any positive real Number, $n \in N$, and f, g are scalar valued polynomials with $f^2 + 8g > 0$. Generalization of k -Jacobsthal sequence as a second order linear recurrence sequence $J_{k,n}$ is

$$J_{k,n} = fJ_{k,n-1} + 2gJ_{k,n-2}, n \geq 2.$$

With initial condition $J_{k,0} = a, J_{k-1} = b$,

MAIN RESULTS

SUMMING FORMULA OF GENERALISED k -JACOBSTHAL NUMBER WITH POSITIVE SUBSCRIPTS

Theorem 1.1: Let $x \in C, n \geq 0$

a. If $2gx^2 + fx - 1 \neq 0$

$$\sum_{p=0}^n x^p J_{k,p} = \frac{x^{n+2} J_{k,n+2} + x^{n+1} (1 - fx) J_{k,n+1} - x J_{k,1} + (fx - 1) J_{k,0}}{2gx^2 + fx - 1}$$

b. If $f^2x - 4g^2x^2 + 4gx - 1 \neq 0$

$$\sum_{p=0}^n x^p J_{k,2p} = \frac{-x^{n+1} (2gx - 1) J_{k,2n+2} + 2fgx^{n+2} J_{k,2n+1} - fx J_{k,1} + (f^2x + 2gx - 1) J_{k,0}}{f^2x - 4g^2x^2 + 4gx - 1}$$

c. If $f^2x - 4g^2x^2 + 4gx - 1 \neq 0$

$$\sum_{p=0}^n x^p J_{k,2p+1} = \frac{fx^{n+1} J_{k,2n+2} - 2gx^{n+1} (2gx - 1) J_{k,2n+1} + (2gx - 1) J_{k,1} - 2fgx J_{k,0}}{f^2x - 4g^2x^2 + 4gx - 1}$$

Proof:

(a)

$$\begin{aligned} J_{k,n} &= fJ_{k,n-1} + 2gJ_{k,n-2} \\ 2gJ_{k,n-2} &= J_{k,n} - fJ_{k,n-1} \\ 2gxJ_{k,1} &= xJ_{k,3} - fJ_{k,2} \\ 2gx^2J_{k,2} &= x^2J_{k,4} - fJ_{k,3} \\ 2gx^{n-1}J_{k,n-1} &= x^{n-1}J_{k,n+1} - fJ_{k,n} \\ 2gx^nJ_{k,n} &= x^nJ_{k,n+2} - fJ_{k,n+1} \end{aligned}$$

Add all the above by side by, we obtain

$$\sum_{p=0}^n x^p J_{k,p} = \frac{x^{n+2}J_{k,n+2} + x^{n+1}(1-fx)J_{k,n+1} - xJ_{k,1} + (fx-1)J_{k,0}}{2gx^2 + fx - 1}$$

(b)

$$\begin{aligned} J_{k,n} &= fJ_{k,n-1} + 2gJ_{k,n-2} \\ fJ_{k,n-1} &= J_{k,n} - 2gJ_{k,n-2} \\ fxJ_{k,3} &= xJ_{k,4} - 2gxJ_{k,2} \\ fx^2J_{k,2} &= x^2J_{k,4} - fJ_{k,3} \\ 2gx^{n-1}J_{k,n-1} &= x^{n-1}J_{k,n+1} - fJ_{k,n} \\ 2gx^nJ_{k,n} &= x^nJ_{k,n+2} - fJ_{k,n+1} \end{aligned}$$

Adding all the above equation by side by we get

$$f \left(-J_{k,1} + \sum_{p=0}^n x^p J_{k,2p+1} \right) = \left(x^n J_{k,2n+2} - J_{k,2} - x^{-1}J_{k,0} + \sum_{p=0}^n x^{p-1}J_{k,2p} \right) - 2g \left(-J_{k,0} + \sum_{p=0}^n x^p J_{k,2p} \right)$$

Similarly

$$\begin{aligned} J_{k,n} &= fJ_{k,n-1} + 2gJ_{k,n-2} \\ fxJ_{k,2} &= xJ_{k,3} - 2gxJ_{k,1} \\ fx^2J_{k,4} &= x^2J_{k,5} - 2gx^2J_{k,3} \\ fx^3J_{k,6} &= x^3J_{k,7} - 2gx^3J_{k,5} \text{ etc ...} \\ fx^{n-1}J_{k,2n-2} &= x^{n-1}J_{k,2n-1} - 2gx^{n-1}J_{k,2n-3} \\ fx^nJ_{k,2n} &= x^nJ_{k,2n+1} - 2gx^nJ_{k,2n-1} \end{aligned}$$

Adding all the above equation by side by we get

$$f(k) \left(-J_{k,0} + \sum_{p=0}^n x^p J_{k,2p} \right) = \left(-J_{k,1} + \sum_{p=0}^n x^p J_{k,2p+1} \right) - 2g \left(-x^{n+1}J_{k,2p+1} + \sum_{p=0}^n x^{p+1}J_{k,2p+1} \right) \quad (3)$$

(2) – (3) we get the result of (b), (c)

Case 1: When $x = 1$ (Substitute $x = 1$ in **Theorem:1** we get the following result.)

Theorem 1.2: For $n \geq 1$

$$\begin{aligned} a) \sum_{p=0}^n J_{k,p} &= \frac{J_{k,n+2} + (1-f)J_{k,n+1} - J_{k,1} + (f-1)J_{k,0}}{2g+f-1} \\ b) \sum_{p=0}^n J_{k,2p} &= \frac{-(2g-1)J_{k,2n+2} + 2fgJ_{k,2n+1} - fJ_{k,1} + (f^2+2g-1)J_{k,0}}{f^2-4g^2+4g-1} \\ c) \sum_{p=0}^n J_{k,2p+1} &= \frac{fJ_{k,2n+2} - 2g(2g-1)J_{k,2n+1} + (2g-1)J_{k,1} - 2fgJ_{k,0}}{f^2-4g^2+4g-1} \end{aligned}$$

Proposition 1.2.1: If $f = k$, $k \in N$, and $g = 1$ with $J_{k,0} = 0, J_{k,1} = 1$ we get sum formula of **k – Jacobsthal numbers**

$$\begin{aligned} a) \sum_{p=0}^n J_{k,p} &= \frac{J_{k,n+2} + (1-k)J_{k,n+1} - 1}{k+1} \\ b) \sum_{p=0}^n J_{k,2p} &= \frac{-J_{k,2n+2} + 2kJ_{k,2n+1} - k}{k^2 - 1} \\ c) \sum_{p=0}^n J_{k,2p+1} &= \frac{kJ_{k,2n+2} - 2J_{k,2n+1} + 1}{k^2 - 1} \end{aligned}$$

Proposition 1.2.2: If $J_{k,n} = \hat{f}_{k,n}$, $f = k$, $k \in N$, and $g = 1$ with $\hat{f}_{k,0} = 2$, $\hat{f}_{k,1} = k$ we get sum formula of **k – Jacobsthal Lucas numbers**

$$\begin{aligned} a) \sum_{p=0}^n \hat{f}_{k,p} &= \frac{\hat{f}_{k,n+2} + (1-k)\hat{f}_{k,n+1} + k - 2}{k+1} \\ b) \sum_{p=0}^n \hat{f}_{k,2p} &= \frac{-\hat{f}_{k,2n+2} + 2k\hat{f}_{k,2n+1} - k^2}{k^2 - 1} \\ c) \sum_{p=0}^n \hat{f}_{k,2p+1} &= \frac{k\hat{f}_{k,2n+2} - 2\hat{f}_{k,2n+1} - 3k}{k^2 - 1} \end{aligned}$$

Proposition 1.2.3: If $J_{k,n} = D\hat{f}_{k,n}$, $f = k$, $k \in N$, and $g = -1$ with $D\hat{f}_{k,0} = 2$, $D\hat{f}_{k,1} = 1$ we get sum formula of **Derived k – Jacobsthal numbers**

$$\begin{aligned} a) \sum_{p=0}^n D\hat{f}_{k,p} &= \frac{D\hat{f}_{k,n+2} + (1-k)D\hat{f}_{k,n+1} - 1}{k-3} \\ b) \sum_{p=0}^n D\hat{f}_{k,2p} &= \frac{3D\hat{f}_{k,2n+2} - 2k\hat{f}_{k,2n+1} - k}{k^2 - 9} \\ c) \sum_{p=0}^n D\hat{f}_{k,2p+1} &= \frac{kD\hat{f}_{k,2n+2} - 6D\hat{f}_{k,2n+1} - 3}{k^2 - 9} \end{aligned}$$

Proposition 1.2.4: Let $J_{k,n} = D\hat{c}_{k,n}$, $f = k$, $k \in N$, and $g = -1$ with $D\hat{c}_{k,0} = 2$, $D\hat{c}_{k,1} = k$ we get sum formula of **Derived k – Jacobsthal Lucas numbers**

$$\begin{aligned} a) \sum_{p=0}^n D\hat{c}_{k,p} &= \frac{D\hat{c}_{k,n+2} + (1-k)D\hat{c}_{k,n+1} + k - 2}{k-3} \\ b) \sum_{p=0}^n D\hat{c}_{k,2p} &= \frac{3D\hat{c}_{k,2n+2} - 2k\hat{c}_{k,2n+1} + k^2 - 6}{k^2 - 9} \\ c) \sum_{p=0}^n D\hat{c}_{k,2p+1} &= \frac{kD\hat{c}_{k,2n+2} - 6D\hat{c}_{k,2n+1} + k}{k^2 - 9} \end{aligned}$$

Case 2: When $x = -1$ (Substitute $x = -1$ in **Theorem:1** we get the following result.)

Theorem 1.3:

a) If $2g - f - 1 \neq 0$ then

$$\sum_{p=0}^n (-1)^p J_{k,p} = \frac{(-1)^n J_{k,n+2} + (-1)^{n+1} (1+f) J_{k,n+1} + J_{k,1} - (f+1) J_{k,0}}{2g - f - 1}$$

b) If $-(f^2) - 4g^2 - 4g - 1 \neq 0$ then

$$\sum_{p=0}^n (-1)^p J_{k,2p} = \frac{(-1)^{n+1} (2g+1) J_{k,2n+2} + (-1)^n 2fg J_{k,2n+1} + f J_{k,1} - (f^2 + 2g + 1) J_{k,0}}{-(f^2 + 4g^2 + 4g + 1)}$$

If $-(f^2) - 4g^2 - 4g - 1 \neq 0$ then

$$\sum_{p=0}^n (-1)^p J_{k,2p+1} = \frac{(-1)^{n+1} f J_{k,2n+2} + (-1)^{n+1} 2g(2g+1) J_{k,2n+1} - (2g+1) J_{k,1} + 2fg J_{k,0}}{-(f^2 + 4g^2 + 4g + 1)}$$

Proposition 1.3.1: If $f = k$, $k \in N$, and $g = 1$ with $J_{k,0} = 0, J_{k,1} = 1$ then we get sum formula of k – Jacobsthal numbers

$$\begin{aligned} a) \sum_{p=0}^n (-1)^p J_{k,p} &= \frac{(-1)^n J_{k,n+2} + (-1)^{n+1} (1+k) J_{k,n+1} + 1}{-k+1} \\ b) \sum_{p=0}^n (-1)^p J_{k,2p} &= \frac{(-1)^{n+1} 3J_{k,2n+2} + (-1)^n 2kJ_{k,2n+1} + k}{-(k^2+9)} \\ c) \sum_{p=0}^n (-1)^p J_{k,2p+1} &= \frac{(-1)^{n+1} kJ_{k,2n+2} + (-1)^{n+1} 6J_{k,2n+1} - 3}{-(k^2+9)} \end{aligned}$$

Proposition 1.3.2: If $J_{k,n} = \hat{J}_{k,n}$, $f = k$, $k \in N$, and $g = 1$ with $\hat{J}_{k,0} = 2$, $\hat{J}_{k,1} = k$ we get sum formula of k – Jacobsthal Lucas numbers

$$\begin{aligned} a) \sum_{p=0}^n (-1)^p \hat{J}_{k,p} &= \frac{(-1)^n \hat{J}_{k,n+2} + (-1)^{n+1} (1+k) \hat{J}_{k,n+1} - k - 2}{-k+1} \\ b) \sum_{p=0}^n (-1)^p \hat{J}_{k,2p} &= \frac{(-1)^{n+1} + 3\hat{J}_{k,n+2} + (-1)^n 2k\hat{J}_{k,2n+1} - k^2 - 6}{-(k^2+9)} \\ c) \sum_{p=0}^n (-1)^p \hat{J}_{k,2p+1} &= \frac{(-1)^{n+1} k\hat{J}_{k,2n+2} + (-1)^{n+1} 6\hat{J}_{k,2n+1} + k}{-(k^2+9)} \end{aligned}$$

Proposition 1.3.3: If $J_{k,n} = D\hat{J}_{k,n}$, $f = k$, $k \in N$, and $g = -1$ with $D\hat{J}_{k,0} = 0$, $D\hat{J}_{k,1} = 1$ we get sum formula of **Derived k – Jacobsthal numbers**

$$\begin{aligned} a) \sum_{p=0}^n (-1)^p D\hat{J}_{k,p} &= \frac{(-1)^n D\hat{J}_{k,n+2} + (-1)^{n+1} (1+k) D\hat{J}_{k,n+1} + 1}{-k-3} \\ b) \sum_{p=0}^n (-1)^p D\hat{J}_{k,2p} &= \frac{(-1)^{n+2} D\hat{J}_{k,2n+2} + (-1)^{n+1} 2k D\hat{J}_{k,2n+1} + k}{-(k^2+1)} \\ c) \sum_{p=0}^n (-1)^p D\hat{J}_{k,2p+1} &= \frac{(-1)^{n+1} k D\hat{J}_{k,2n+2} + 2(-1)^{n+1} D\hat{J}_{k,2n+1} + 1}{-(k^2+1)} \end{aligned}$$

Proposition 1.3.4: Let $J_{k,n} = D\hat{c}_{k,n}$, $f = k$, $k \in N$, and $g = -1$ with $D\hat{c}_{k,0} = 2$, $D\hat{c}_{k,1} = k$ we get sum formula of **Derived k – Jacobsthal Lucas numbers**

$$\begin{aligned} a) \sum_{p=0}^n (-1)^p D\hat{c}_{k,p} &= \frac{(-1)^n D\hat{c}_{k,n+2} + (-1)^{n+1} (1+k) D\hat{c}_{k,n+1} + 3k + 2}{-(k+3)} \\ b) \sum_{p=0}^n (-1)^p D\hat{c}_{k,2p} &= \frac{(-1)^{n+2} D\hat{c}_{k,2n+2} + (-1)^{n+1} 2k D\hat{c}_{k,2n+1} - k^2 + 2}{-(k^2+1)} \\ c) \sum_{p=0}^n (-1)^p D\hat{c}_{k,2p+1} &= \frac{(-1)^{n+1} k D\hat{c}_{k,2n+2} + (-1)^{n+1} D\hat{c}_{k,2n+1} - 3k}{-(k^2+1)} \end{aligned}$$

Case 3: When $x = 1 + i$ (Substitute $x = 1 + i$ in **Theorem:1** we get the following result.)

Theorem 1.4:

a) If $2g(1+i)^2 + f(1+i) - 1 \neq 0$ then

$$\sum_{p=0}^n (1+i)^p J_{k,p} = \frac{(1+i)^{n+2} J_{k,n+2} + (1+i)^{n+1} (1-f(1+i)) J_{k,n+1} - (1+i) J_{k,1} + (f(1+i) - 1) J_{k,0}}{2g(1+i)^2 + f(1+i) - 1}$$

b) If $f^2(1+i) - 4g^2(1+i)^2 + 4g(1+i) - 1 \neq 0$ then

$$\begin{aligned} & -(1+i)^{n+1} (2g(1+i) - 1) J_{k,2n+2} + (1+i)^{n+2} 2fg J_{k,2n+1} - f(1+i) J_{k,1} + \\ & \sum_{p=0}^n (1+i)^p J_{k,2p} = \frac{((f^2 + 2g - 1) + (f^2 + 2g)i) J_{k,0}}{f^2(1+i) - 4g^2(1+i)^2 + 4g(1+i) - 1} \end{aligned}$$

c) If $(f^2(1+i) - 4g^2(1+i)^2 + 4g(1+i) - 1) \neq 0$ then

$$\begin{aligned} & \sum_{p=0}^n (1+i)^p J_{k,2p+1} \\ & = \frac{(1+i)^{n+1} f J_{k,2n+2} - 2g(1+i)^{n+1} (2g(1+i) - 1) J_{k,2n+1} + (2g(1+i) - 1) J_{k,1} - 2fg(1+i) J_{k,0}}{f^2(1+i) - 4g^2(1+i)^2 + 4g(1+i) - 1} \end{aligned}$$

Proposition 1.4.1: If $f = k$, $k \in N$, and $g = 1$ with $J_{k,0} = 0, J_{k,1} = 1$ then we get sum formula of k - Jacobsthal numbers

a) If $2(1+i)^2 + k(1+i) - 1 \neq 0$ then

$$\sum_{p=0}^n (1+i)^p J_{k,p} = \frac{(1+i)^{n+2} J_{k,n+2} + (1+i)^{n+1} (1-k(1+i)) J_{k,n+1} - (1+i)}{2(1+i)^2 + k(1+i) - 1}$$

b) If $(k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p J_{k,2p} = \frac{-(1+i)^{n+1} (1+2i) J_{k,2n+2} + (1+i)^{n+2} 2k J_{k,2n+1} - k(1+i)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1}$$

c) If $(k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p J_{k,2p+1} = \frac{(1+i)^{n+1} k J_{k,2n+2} - 2(1+i)^{n+1} (1+2i) J_{k,2n+1} + (1+2i)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1}$$

Proposition 1.4.2: If $J_{k,n} = \hat{f}_{k,n}$, $f = k$, $k \in N$, and $g = 1$ with $\hat{f}_{k,0} = 2$, $\hat{f}_{k,1} = k$ we get sum formula of k - Jacobsthal Lucas numbers

a) If $2(1+i)^2 + k(1+i) - 1 \neq 0$ then

$$\sum_{p=0}^n (1+i)^p \hat{f}_{k,p} = \frac{(1+i)^{n+2} \hat{f}_{k,n+2} + (1+i)^{n+1} (1-k(1+i)) \hat{f}_{k,n+1} - (1+i)k + ((2k-2) + i2k)}{2(1+i)^2 + k(1+i) - 1}$$

b) If $(k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1) \neq 0$ then

$$\begin{aligned} & \sum_{p=0}^n (1+i)^p \hat{f}_{k,2p} \\ & = \frac{-(1+i)^{n+1} (1+2i) \hat{f}_{k,2n+2} + (1+i)^{n+2} 2k \hat{f}_{k,2n+1} - k^2(1+i) + 2(k^2(1+i) + 2(1+i) - 1)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1} \end{aligned}$$

c) If $(k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p \hat{f}_{k,2p+1} = \frac{(1+i)^{n+1} k \hat{f}_{k,2n+2} - 2(1+i)^{n+1} (1+2i) \hat{f}_{k,2n+1} + (1+2i)k - 4k(1+i)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1}$$

Proposition 1.4.3: Let $J_{k,n} = D\hat{f}_{k,n}, f = k, k \in N$, and $g = -1$ with $D\hat{f}_{k,0} = 0, D\hat{f}_{k,1} = 1$ we get sum formula of **Derived k - Jacobsthal numbers**

a) If $-2(1+i)^2 + k(1+i) - 1 \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{f}_{k,p} = \frac{(1+i)^{n+2} D\hat{f}_{k,n+2} + (1+i)^{n+1} (1 - k(1+i)) D\hat{f}_{k,n+1} - (1+i)}{-2(1+i)^2 + k(1+i) - 1}$$

b) If $(k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{f}_{k,2p} = \frac{-(1+i)^{n+1} (-3 - 2i) D\hat{f}_{k,2n+2} - (1+i)^{n+2} 2k D\hat{f}_{k,2n+1} - k(1+i)}{k^2(1+i) - 4(1+i)^2 - 4(1+i) - 1}$$

c) If $(k^2(1+i) - 4(1+i)^2 - 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{f}_{k,2p+1} = \frac{(1+i)^{n+1} k D\hat{f}_{k,2n+2} + 2(1+i)^{n+1} (-3 - 2i) D\hat{f}_{k,2n+1} - (3 + 2i)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1}$$

Proposition 1.4.4: Let $J_{k,n} = D\hat{c}_{k,n}, f = k, k \in N$, and $g = -1$ with $D\hat{c}_{k,0} = 2, D\hat{c}_{k,1} = k$ we get sum formula of **Derived k - Jacobsthal Lucas numbers**

a) If $-2(1+i)^2 + k(1+i) - 1 \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{c}_{k,p} = \frac{(1+i)^{n+2} D\hat{c}_{k,n+2} + (1+i)^{n+1} (1 - k(1+i)) D\hat{c}_{k,n+1} - (1+i)k + 2(k(1+i) - 1)}{-2(1+i)^2 + k(1+i) - 1}$$

b) If $(k^2(1+i) - 4(1+i)^2 - 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{c}_{k,2p} = \frac{-(1+i)^{n+1} (-2(1+i) - 1) D\hat{c}_{k,2n+2} - (1+i)^{n+2} 2k D\hat{c}_{k,2n+1} - k^2(1+i)}{k^2(1+i) - 4(1+i)^2 - 4(1+i) - 1}$$

c) If $(k^2(1+i) - 4(1+i)^2 - 4(1+i) - 1) \neq 0$ then

$$\sum_{p=0}^n (1+i)^p D\hat{c}_{k,2p+1} = \frac{(1+i)^{n+1} k D\hat{c}_{k,2n+2} + 2(1+i)^{n+1} (-2(1+i) - 1) D\hat{c}_{k,2n+1} + k(-2(1+i) - 1) + 4k(1+i)}{k^2(1+i) - 4(1+i)^2 + 4(1+i) - 1}$$

CONCLUSION

We discussed Sum formula for Generalised k - Jacobsthal Numbers for three cases. From this we deduced Sum formula for k - Jacobsthal, k - Jacobsthal Lucas, Derived k - Jacobsthal, Derived k - Jacobsthal Lucas.

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